Opened for Result Evaluation Purpose.
02/07/2017

Total No. of Printed Pages : 17

(DO NOT OPEN THIS QUESTION BOOKLET BEFORE TIME OR UNTIL YOU ARE ASKED TO DO SO)

PG-EE-2017

SUBJECT : Mathematics Hons. (Five Year)

A

10005

Sr. No. ....

Time : **1¼ Hours** | Total Questions : **100** | Max. Marks : **100**

Roll No. (in figures)________________ (in words)________________________________

Name ____ Date of Birth ________________

Father's Name ________________________ Mother's Name ________________________

Date of Exam ____________________

SEAL

____
(Signature of the Candidate)

____
(Signature of the Invigilator)

CANDIDATES MUST READ THE FOLLOWING INFORMATION/INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. ***All questions are compulsory and carry equal marks. The candidates are required to attempt all questions.***
2. The candidates ***must return*** the question booklet as well as OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means/misbehaviour will be registered against him/her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. In case there is any discrepancy in any question(s) in the Question Booklet, the same may be brought to the notice of the Controller of Examinations in writing **within two hours** after the test is over. No such complaint(s) will be entertained thereafter.
4. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
5. ***Use only black or blue ball point pen of good quality in the OMR Answer-Sheet.***
6. There will be ***no negative*** marking. Each correct answer will be awarded ***one*** full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.
7. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. If A, B and C are three sets and X is the universal set such that $n(X) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Then $n(A' \cap B') =$

(1) 200 (2) 300 (3) 400 (4) 500

2. If $A = \{4^n - 3n - 1 \mid n \in N\}$ and $B = \{9(n-1) \mid n \in N\}$, then $A \cup B =$

(1) N (2) A (3) B (4) $B - A$

3. Let A and B be two non-empty subsets of a set X such that A is not a subset of B, then which of the following is true ?

(1) B is a subset of A

(2) A and B are disjoint

(3) A is a subset of complement of B

(4) A and complement of B are non-disjoint

4. If A and B are two non-empty sets having n elements in common, then the number of elements in common in $A \times B$ and $B \times A$ are :

(1) $2n$ (2) n^2 (3) n^n (4) 2^n

5. A relation R defined from $\{2, 3, 4, 5\}$ to $\{3, 6, 7, 10\}$ by $xRy \Leftrightarrow x$ is relatively prime to y. Then domain of R is :

(1) $\{3, 5\}$ (2) $\{2, 3, 5\}$ (3) $\{2, 3, 4\}$ (4) $\{2, 3, 4, 5\}$

6. Which of the following is a function ?

(1) $\{(x, y) : y = |x|, x, y \in R\}$ (2) $\{(x, y) : x = y^2, x, y \in R\}$

(3) $\{(x, y) : x^2 - y^2 = 1, x, y \in R\}$ (4) $\{(x, y) : x^2 + y^2 = 1, x, y \in R\}$

7. In a triangle ABC, right angled at C and having sides a, b, c, $\tan A + \tan B =$

(1) $\frac{a^2}{bc}$ (2) $\frac{c^2}{ab}$ (3) $\frac{b^2}{ac}$ (4) $\frac{a+b}{c}$

8. If A lies in the second quadrant and $3\tan A + 4 = 0$, then $2\cot A - 5\cos A + \sin A =$

(1) $\frac{5}{3}$ (2) $\frac{7}{10}$ (3) $\frac{23}{10}$ (4) $\frac{37}{10}$

9. If A and B are acute positive angles satisfying the equations $3\sin^2 A + 2\sin^2 B = 1$ and $3\sin 2A - 2\sin 2B = 0$, then $A + 2B =$

(1) 0 (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

10. If $\sin 2\theta = \cos 3\theta$ and θ is an acute angle, then $\sin\theta =$

(1) $\frac{\sqrt{5}-1}{4}$ (2) $\frac{\sqrt{5}+1}{4}$ (3) $\frac{\sqrt{3}-1}{2}$ (4) $\frac{\sqrt{3}-1}{4}$

11. If α is an acute angle and $\sin\frac{\alpha}{2} = \frac{\sqrt{x-1}}{2x}$, than $\tan\alpha =$

(1) $\sqrt{\frac{x-1}{x+1}}$ (2) $\frac{\sqrt{x-1}}{x+1}$ (3) $\sqrt{1-x^2}$ (4) $\sqrt{x^2-1}$

12. The statement $P(n)$: "$2 + 4 + 6 + \ldots\ldots + 2n = n(n+1) + 2$" is given true for $n = k$, then for $n = k + 1$, it is :

(1) not defined (2) true (3) not true (4) meaningless

13. If $iz^3 + z^2 - z + i = 0$, then $|z| =$

(1) 4 (2) 3 (3) 2 (4) 1

SEAL

14. If $\left|z-\frac{4}{z}\right|=2$, then the greatest value of $|z|$ is :

(1) $2+\sqrt{2}$ (2) $\sqrt{3}+1$ (3) $\sqrt{5}+1$ (4) $\sqrt{5}-1$

15. Solution of the inequality $\frac{x+1}{x+2}\geq 1$

(1) $(-\infty,-1)$ (2) $(-\infty,-2)$ (3) $(-1,\infty)$ (4) $(2,\infty)$

16. If $0<r<s\leq n$ and ${}^nP_r={}^nP_s$, then $r+s=$

(1) $2n-1$ (2) $n-2$ (3) $2n$ (4) 2

17. The number of words that can be formed by using the letters of the word MATHEMATICS that start as well as end with A, is :

(1) 37528 (2) 45360 (3) 90720 (4) 362880

18. A polygon has 44 diagonals. The number of its sides are :

(1) 44 (2) 22 (3) 11 (4) 9

19. The expression $P(x)=\left(\sqrt{x^5-1}+x\right)^7-\left(\sqrt{x^5-1}-x\right)^7$ is a polynomial of degree :

(1) 14 (2) 16 (3) 17 (4) 18

20. The remainder when 2^{2003} is divided by 17 is :

(1) 2 (2) 4 (3) 7 (4) 8

21. Let x be the arithmetic mean and y, z be the two geometric means between any two positive numbers. Then $\frac{y^3+z^3}{xyz}=$

(1) 4 (2) 3 (3) 2 (4) $\frac{3}{2}$

22. If $1 + 6 + 11 + 16 + \ldots\ldots + x = 148$, then $x =$

(1) 31 (2) 26 (3) 41 (4) 36

23. If each term of an infinite G. P. is twice the sum of the terms following it, then the common ratio of the G. P. is :

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$ (3) $\frac{2}{3}$ (4) $\frac{3}{4}$

24. A line joining two points $A(2, 0)$ and $B(3, 1)$ is rotated about A in anticlockwise direction through an angle 15°. The equation of the line in the new position is :

(1) $\sqrt{3}x + y = 2\sqrt{3}$ (2) $\sqrt{3}x - y = 2\sqrt{3}$

(3) $\sqrt{3}y + x = 2\sqrt{3}$ (4) $\sqrt{3}y - x = 2\sqrt{3}$

25. If p_1 and p_2 denote the lengths of the perpendiculars from the origin on the lines $x \sec\alpha + y \operatorname{cosec}\alpha = 2a$ and $x \cos\alpha + y \sin\alpha = a \cos 2\alpha$ respectively, then $\left(\frac{p_1}{p_2} + \frac{p_2}{p_1}\right)^2 =$

(1) $4\operatorname{cosec}^2 4\alpha$ (2) $4\sec^2 4\alpha$ (3) $2\operatorname{cosec}^2 4\alpha$ (4) $2\cos^2 4\alpha$

26. A line is drawn through the point $P(4, 11)$ to cut the circle $x^2 + y^2 = 9$ at the points A and B. Then $PA \cdot PB =$

(1) 9 (2) 121 (3) 128 (4) 139

27. The focus of the parabola $(y-1)^2 = 12(x-2)$ is :

(1) (5, 1) (2) (1, 5) (3) (2, 1) (4) (3, 0)

28. In an ellipse, if the lines joining a focus to the extremities of the minor axis make an equilateral triangle with the minor axis, the eccentricity of the ellipse is :

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{\sqrt{3}}{2}$

29. The number of lines in three dimensions which are equally inclined to the co-ordinate axes is :

(1) 8 (2) 6 (3) 4 (4) 3

30. $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} =$

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{1}{2}$ (4) 0

31. $\lim_{x \to 2} \frac{x-2}{|x-2|} =$

(1) 0 (2) -2 (3) 1 (4) limit does not exist

32. Let $f(x + y) = f(x)\,f(y)$ for all x and y. If $f(5) = 2$ and $f'(0) = 3$, then $f'(5) =$

(1) 6 (2) 5 (3) $\frac{3}{2}$ (4) $\frac{2}{3}$

33. Let $f(x) = \begin{cases} x^2, & x \geq 1 \\ ax + b, & x < 1 \end{cases}$. If f is a differentiable function, then :

(1) $a = -1, b = 2$ (2) $a = 2, b = -1$

(3) $a = -\frac{1}{2}, b = \frac{3}{2}$ (4) $a = \frac{1}{2}, b = \frac{1}{2}$

34. Given the statements :

p : All composite numbers are even numbers.

q : All composite numbers are odd numbers.

Then :

(1) both p and q are true (2) p is true, q is false

(3) q is true, p is false (4) both p and q are false

35. The sum of squares of deviations of a set of values is minimum when taken about :

(1) mode
(2) median
(3) geometric mean
(4) arithmetic mean

36. If each observation of a raw data whose variance is σ^2 is multiplied by K, then the variance of the new set is :

(1) $K^2\sigma^2$
(2) $K\sigma^2$
(3) σ^2
(4) $K^2 + \sigma^2$

37. A group of 6 boys and 6 girls is randomly divided into two equal groups. The probability that each group contains 3 boys and 3 girls is :

(1) $\frac{90}{231}$
(2) $\frac{100}{231}$
(3) $\frac{110}{231}$
(4) $\frac{36}{231}$

38. In three throws of a pair of dice, the probability of throwing doublets not more than twice is :

(1) $\frac{5}{72}$
(2) $\frac{211}{216}$
(3) $\frac{35}{36}$
(4) $\frac{215}{216}$

39. The probability that a teacher will give an unanounced test during any class meeting is $\frac{1}{5}$. If a student is absent twice, the probability that he will miss atleast one test, is :

(1) $\frac{2}{25}$
(2) $\frac{7}{25}$
(3) $\frac{9}{25}$
(4) $\frac{16}{25}$

40. The probability that the 13th day of a randomly chosen month is a second saturday, is :

(1) $\frac{19}{84}$
(2) $\frac{1}{84}$
(3) $\frac{1}{7}$
(4) $\frac{1}{12}$

41. If $f : X \to Y$ defined by $f(x) = \sqrt{3}\sin x + \cos x + 4$ is one-one and onto, then Y is :

(1) $[2, 4]$
(2) $[1, 5]$
(3) $[2, 5]$
(4) $[2, 6]$

42. Inverse of the function $f(x) = \sin^{-1}\left\{4-(x-7)^3\right\}^{1/5}$ is :

(1) $7+\left(4-\sin^5 x\right)^{1/3}$ (2) $7+\left(4+\sin^5 x\right)^{1/3}$

(3) $7-\left(4-\sin^5 x\right)^{1/3}$ (4) $\left(4-\sin^5 x\right)^{1/3}$

43. For real numbers x and y, we write $xRy \Leftrightarrow x-y+\sqrt{2}$ is an irrational number. Then the relation R is :

(1) reflexive (2) symmetric (3) transitive (4) equivalence

44. If $f(x) = \left(a-x^n\right)^{1/n}$, where $a > 0$ and $n \in N$, then $fof(x) =$

(1) a (2) x (3) a^n (4) x^n

45. Value of $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$ is equal to :

(1) $\frac{3}{4}$ (2) $-\frac{1}{4}$ (3) $\frac{5}{8}$ (4) $\frac{1}{16}$

46. If $4\sin^{-1}x + \cos^{-1}x = \pi$, then $x =$

(1) $\frac{3}{4}$ (2) $-\frac{1}{2}$ (3) $\frac{1}{2}$ (4) 1

47. $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)=$

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) $\frac{3\pi}{4}$

48. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is :

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{\pi}{3}$ (4) $\frac{4\pi}{3}$

49. If w is the complex cube root of unity and $A=\begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix}$, then $A^{40}=$

(1) 0
(2) 1
(3) A
(4) $-A$

50. The inverse of a skew symmetric matrix of odd order is :

(1) diagonal matrix
(2) symmetric matrix
(3) skew-symmetric matrix
(4) inverse does not exist

51. If $A(\theta)=\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$ and $AB=I$, then $(\sec^2\theta)B=$

(1) $A(\theta)$
(2) $A(-\theta)$
(3) $-A(\theta)$
(4) $A(\theta/2)$

52. If the matrix $A=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & a \end{bmatrix}$ is unitary, then the value of a is :

(1) -1
(2) 0
(3) -2
(4) 1

53. If $f(\theta)=\begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$, then the value of $f\left(\frac{5\pi}{3}\right)=$

(1) 0
(2) 1
(3) -1
(4) $\frac{\sqrt{3}}{2}$

54. The complex number $z=\begin{vmatrix} 2 & 3+i & -3 \\ 3-i & 0 & -1+i \\ -3 & -1-i & 4 \end{vmatrix}$ is equal to :

(1) $2-5i$
(2) $3-4i$
(3) $5+4i$
(4) None of these

55. If the system of linear equations $x+y+z=6$, $x+2y+3z=4$, $2x+5y+\lambda z=k$ has a unique solution, then :

(1) $\lambda=8, k=36$
(2) $\lambda=8, k\neq 36$
(3) $\lambda\neq 8$
(4) $\lambda=8, k=24$

56. If x, y, z are all distinct and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then the value of xyz is :

(1) -1 (2) -2 (3) $-\frac{1}{2}$ (4) 1

57. If B is a non-singular matrix and A is square matrix, then $\text{Det}\left(B^{-1}AB\right) =$

(1) $\text{Det}(A)$ (2) $\text{Det}(B)$ (3) $\text{Det}\left(A^{-1}\right)$ (4) $\text{Det}\left(B^{-1}\right)$

58. If A is skew-symmetric matrix and n is an even positive integer, then A^n is :

(1) skew-symmetric matrix (2) symmetric matrix

(3) diagonal matrix (4) unitary matrix

59. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2 =$

(1) $2AB$ (2) AB (3) $A + B$ (4) $2(A + B)$

60. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x) =$

(1) $ax(3x + 2a)$ (2) $a(3x + 2a)$ (3) $ax(2x + 3a)$ (4) $x(3x + 2a)$

61. If $f(x) = \dfrac{3x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at each point in its domain, then the value of $f(0)$ is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{4}{3}$

62. The number of points of discontinuity for $f(x) = \dfrac{1}{\log|x|}$, is :

(1) 2 (2) 3 (3) 4 (4) 1

63. If $f(x)=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then $f'(2)=$

(1) $\frac{2}{5}$ (2) $\frac{3}{10}$ (3) $\frac{1}{8}$ (4) $\frac{1}{10}$

64. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w. r. t. $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=0$, is :

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) $\frac{1}{2}$ (4) 1

65. If $x^y=e^{x-y}$, then $\frac{dy}{dx}=$

(1) $\frac{x^2}{(1+\log x)^2}$ (2) $\frac{\log x}{1+\log x}$ (3) $\frac{\log x}{(1+\log x)^2}$ (4) $\frac{x}{(1+\log x)^2}$

66. If $f(x)=x^2e^{-x}$, then the interval in which $f(x)$ increases with respect to x, is :

(1) $(0, 1)$ (2) $(-2, 0)$ (3) $(0, 2)$ (4) $(2, \infty)$

67. The normal to the curve $y = f(x)$ at the point $(3, 4)$ makes an angle $\frac{3\pi}{4}$ with the positive x-axis, then $f'(3)=$

(1) $\frac{3}{4}$ (2) $-\frac{3}{4}$ (3) -1 (4) 1

68. If $f(x) = x(x-2)(x-4)$, $1 \le x \le 4$, then a number satisfying the conditions of the Lagrange's mean value theorem is :

(1) 3 (2) $\frac{4}{3}$ (3) $\frac{5}{2}$ (4) 2

69. If x and y are two real numbers such that $x > 0$ and $xy = 1$, then the minimum value of $x + y$ is :

(1) $\frac{3}{2}$ (2) $\frac{1}{4}$ (3) 1 (4) 2

70. The critical points of the function $f(x) = (2x+1)(x-2)^{2/3}$ are :

(1) 1 and 2 (2) -1 and 2 (3) $-\frac{1}{2}$ and 2 (4) $\frac{1}{2}$ and 2

71. $\int \frac{dx}{\sqrt{(1-x)(x-2)}} =$

(1) $\sin^{-1}(2x-5)+c$ (2) $\sin^{-1}(3-2x)+c$

(3) $\sin^{-1}(2x-3)+c$ (4) $\sin^{-1}(2x+5)+c$

72. $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx =$

(1) $\sqrt{2}\sin^{-1}(\sin x-\cos x)+c$ (2) $\sqrt{2}\sin^{-1}(\sin x+\cos x)+c$

(3) $\sqrt{2}\cos^{-1}(\sin x-\cos x)+c$ (4) $\sqrt{2}\cos^{-1}(\sin x+\cos x)+c$

73. $\int \frac{\cos 2x - \cos 2\alpha}{\sin x - \sin \alpha}dx =$

(1) $2\sin x - 2x\cos\alpha + c$ (2) $2\cos x - 2x\sin\alpha + c$

(3) $2\sin x - x\cos\alpha + c$ (4) $2\cos x - x\sin\alpha + c$

74. $\int \frac{xe^x}{(1+x)^2}dx =$

(1) $\frac{e^x}{(1+x)^2}+c$ (2) $\frac{e^x}{x+1}+c$ (3) $\frac{-e^x}{x+1}+c$ (4) $\frac{-e^x}{(1+x)^2}+c$

75. Value of $\int_0^{2\pi} \frac{dx}{e^{\sin x}+1}$ is :

(1) 0 (2) $\frac{\pi}{2}$ (3) 2π (4) π

76. Value of $\int_{1/e}^{e} |\log x|\, dx$ is :

(1) $2\left(1-\frac{1}{e}\right)$ (2) $2\left(\frac{1}{e}-1\right)$ (3) $1-\frac{1}{e}$ (4) $\frac{1}{e}-1$

77. The area bounded by $y = x^2$, $y = [x+1]$, $0 \le x < 1$ and the y-axis, where [.] denotes the greatest integer function, is :

(1) $\frac{4}{3}$ sq. units (2) $\frac{1}{3}$ sq. units (3) $\frac{2}{3}$ sq. units (4) $\frac{1}{2}$ sq. units

78. If $I = \int_0^{\pi/2} \frac{dx}{5+3\cos x} = \lambda \tan^{-1}\left(\frac{1}{2}\right)$, then $\lambda =$

(1) $\frac{1}{4}$ (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) 1

79. Area bounded by the parabola $y^2 = 4ax$ and its latus rectum is :

(1) $\frac{2}{3}a^2$ (2) $\frac{5}{8}a^2$ (3) $\frac{4}{3}a^2$ (4) $\frac{8}{3}a^2$

80. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$, the line $x = \sqrt{3}y$ and x-axis is :

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{2\pi}{3}$

81. The degree and order of the differential equation of all parabolas whose axis is x-axis, are :

(1) 2, 1 (2) 1, 1 (3) 1, 2 (4) 3, 2

82. A particular solution of $\log\left(\frac{dy}{dx}\right) = 3x + 4y$, $y(0) = 0$, is :

(1) $4e^{3x} - 3e^{-4y} = 12$
(2) $4e^{3x} - 3e^{-4y} = 7$
(3) $4e^{3x} + 3e^{-4y} = 12$
(4) $4e^{3x} + 3e^{-4y} = 7$

83. The solution of the equation $\frac{dy}{dx} = \cos(x - y)$ is :

(1) $x + \cot\left(\frac{x-y}{2}\right) + c$
(2) $x + \tan\left(\frac{x-y}{2}\right) + c$
(3) $y + \cot\left(\frac{x-y}{2}\right) + c$
(4) $y + \tan\left(\frac{x-y}{2}\right) + c$

84. The integrating factor of the differential equation $x \log x \frac{dy}{dx} + y = 2 \log x$, is :

(1) $\log(\log x)$ (2) $\log x$ (3) $-\log x$ (4) e^x

85. A unit vector at $t = 2$ on the curve $x = t^2 + 2$, $y = 4t - 5$, $z = 2t^2 - 6t$, is :

(1) $\frac{1}{3}(2\hat{i} + 2\hat{j} + \hat{k})$
(2) $\frac{1}{3}(2\hat{i} + 2\hat{j} - \hat{k})$
(3) $\frac{1}{\sqrt{3}}(2\hat{i} + 2\hat{j} + \hat{k})$
(4) $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$

86. If $\vec{a}$ and $\vec{b}$ are two unit vectors, then the vector $(\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})$ is parallel :

(1) $\vec{a} + \vec{b}$ (2) $2\vec{a} + \vec{b}$ (3) $\vec{a} - \vec{b}$ (4) $2\vec{a} - \vec{b}$

SEAL

87. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the three non-coplanar vectors such that $\vec{a}+\vec{b}+\vec{c}=\alpha\vec{d}$ and $\vec{b}+\vec{c}+\vec{d}=\beta\vec{a}$, then $\vec{a}+\vec{b}+\vec{c}+\vec{d}=$

(1) $\vec{0}$ (2) $\alpha\vec{a}$ (3) $\beta\vec{b}$ (4) $(\alpha+\beta)\vec{c}$

88. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then $\left|\frac{\vec{a}-\vec{b}}{2}\right|=$

(1) $2\sin\theta$ (2) $\sin\theta$ (3) $\sin\frac{\theta}{2}$ (4) $2\sin\frac{\theta}{2}$

89. The lines whose vector equations are $\vec{r}=2\hat{i}-3\hat{j}+7\hat{k}+\lambda(2\hat{i}+a\hat{j}+5\hat{k})$ and $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\mu(3\hat{i}-a\hat{j}+a\hat{k})$ are perpendicular for all values of λ and μ if $a=$

(1) 2 (2) 3 (3) 4 (4) 6

90. The number of lines in three dimensions which are equally inclined to the coordinate axes, is :

(1) 2 (2) 4 (3) 6 (4) 8

91. The equation of a plane which passes through (1, –2, 1) and is perpendicular to two planes $2x-2y+z=0$ and $x-y+2z=4$, is :

(1) $x+2y+3=0$ (2) $3x+2y+1=0$

(3) $x-y-3=0$ (4) $x+y+1=0$

92. The foot of the perpendicular for the point (1, 0, 2) to the line $\frac{x+1}{3}=\frac{y-2}{-2}=\frac{z+1}{-1}$ is the point :

(1) $\left(\frac{1}{2},-1,\frac{3}{2}\right)$ (2) $\left(\frac{1}{2},1,\frac{3}{2}\right)$ (3) $\left(\frac{1}{2},1,\frac{-3}{2}\right)$ (4) $(1,2,-3)$

93. Which of the following sets is ***not*** convex ?

(1) $\{(x, y) \mid 3x^2 + 2y^2 \leq 6\}$ (2) $\{(x, y) \mid 3 \leq x^2 + y^2 \leq 5\}$

(3) $\{(x, y) \mid x \geq 2, x \leq 3\}$ (4) $\{(x, y) \mid y^2 \leq x\}$

94. If the constraints in a linear programming problem are changed, then :

(1) the change in constraints is ignored

(2) solution is not defined

(3) the problem is to be re-evaluated

(4) the objective function has to be modified

95. Which of the following statements is ***correct*** ?

(1) Every LLP admits an optimal solution

(2) A LLP admits a unique optimal solution

(3) The set of all feasible solutions of a LLP is not a convex set

(4) If a LLP admits two optimal solutions then it has an infinite number of optimal solutions

96. If the mean and variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value at most 3 is :

(1) $\frac{4}{5}$ (2) $\frac{7}{8}$ (3) $\frac{15}{16}$ (4) $\frac{13}{16}$

97. A fair coin is tossed n times. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then the value of n is :

(1) 14 (2) 16 (3) 24 (4) 48

98. A letter is known to have come from either CALCUTTA or TATANAGAR. On the envelope, just two consecutive alphabets TA are visible. The probability that letter has come from CALCUTTA is :

(1) $\frac{5}{12}$ (2) $\frac{4}{11}$ (3) $\frac{4}{7}$ (4) $\frac{1}{3}$

99. Three players A, B, C in this order, cut a pack of cards, and the whole pack is reshuffled after each cut. If the winner is one who first draws a diamond, then C's chance of winning is :

(1) $\frac{27}{64}$ (2) $\frac{9}{64}$ (3) $\frac{9}{28}$ (4) $\frac{9}{37}$

100. If two events A and B are such that $P(A) > 0$ and $P(B) \neq 1$, then $P(\overline{A}/\overline{B})$ is equal to :

(1) $\frac{1-P(A\cup B)}{P(\overline{B})}$ (2) $\frac{1-P(A\cap B)}{P(\overline{B})}$ (3) $1-P(\overline{A}/B)$ (4) $1-P(A/B)$

Opened for Result Evaluation Purpose Ektam 02/07/2017

Total No. of Printed Pages : 17

(DO NOT OPEN THIS QUESTION BOOKLET BEFORE TIME OR UNTIL YOU ARE ASKED TO DO SO)

PG-EE-2017

SUBJECT : Mathematics Hons. (Five Year)

B

Sr. No. 10010

Time : 1¼ Hours | Total Questions : 100 | Max. Marks : 100

Roll No. (in figures)____________ (in words) ____________

Name ____________ Date of Birth ____________

Father's Name ____________ Mother's Name ____________

Date of Exam ____________

(Signature of the Candidate) (Signature of the Invigilator)

SEAL

CANDIDATES MUST READ THE FOLLOWING INFORMATION/INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. ***All questions are compulsory and carry equal marks. The candidates are required to attempt all questions.***
2. The candidates ***must return*** the question booklet as well as OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means/misbehaviour will be registered against him/her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. In case there is any discrepancy in any question(s) in the Question Booklet, the same may be brought to the notice of the Controller of Examinations in writing **within two hours** after the test is over. No such complaint(s) will be entertained thereafter.
4. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
5. ***Use only black or blue ball point pen of good quality in the OMR Answer-Sheet.***
6. There will be ***no negative*** marking. Each correct answer will be awarded ***one*** full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.
7. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. The equation of a plane which passes through (1, –2, 1) and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, is :

   (1) $x + 2y + 3 = 0$ (2) $3x + 2y + 1 = 0$

   (3) $x - y - 3 = 0$ (4) $x + y + 1 = 0$

2. The foot of the perpendicular for the point (1, 0, 2) to the line $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z+1}{-1}$ is the point :

   (1) $\left(\frac{1}{2}, -1, \frac{3}{2}\right)$ (2) $\left(\frac{1}{2}, 1, \frac{3}{2}\right)$ (3) $\left(\frac{1}{2}, 1, \frac{-3}{2}\right)$ (4) $(1, 2, -3)$

3. Which of the following sets is ***not*** convex ?

   (1) $\{(x, y) \mid 3x^2 + 2y^2 \leq 6\}$ (2) $\{(x, y) \mid 3 \leq x^2 + y^2 \leq 5\}$

   (3) $\{(x, y) \mid x \geq 2, x \leq 3\}$ (4) $\{(x, y) \mid y^2 \leq x\}$

4. If the constraints in a linear programming problem are changed, then :

   (1) the change in constraints is ignored

   (2) solution is not defined

   (3) the problem is to be re-evaluated

   (4) the objective function has to be modified

5. Which of the following statements is ***correct*** ?

   (1) Every LLP admits an optimal solution

   (2) A LLP admits a unique optimal solution

   (3) The set of all feasible solutions of a LLP is not a convex set

   (4) If a LLP admits two optimal solutions then it has an infinite number of optimal solutions

6. If the mean and variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value at most 3 is :

(1) $\frac{4}{5}$ (2) $\frac{7}{8}$ (3) $\frac{15}{16}$ (4) $\frac{13}{16}$

7. A fair coin is tossed n times. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then the value of n is :

(1) 14 (2) 16 (3) 24 (4) 48

8. A letter is known to have come from either CALCUTTA or TATANAGAR. On the envelope, just two consecutive alphabets TA are visible. The probability that letter has come from CALCUTTA is :

(1) $\frac{5}{12}$ (2) $\frac{4}{11}$ (3) $\frac{4}{7}$ (4) $\frac{1}{3}$

9. Three players A, B, C in this order, cut a pack of cards, and the whole pack is reshuffled after each cut. If the winner is one who first draws a diamond, then C's chance of winning is :

(1) $\frac{27}{64}$ (2) $\frac{9}{64}$ (3) $\frac{9}{28}$ (4) $\frac{9}{37}$

10. If two events A and B are such that $P(A) > 0$ and $P(B) \neq 1$, then $P(\overline{A}/\overline{B})$ is equal to :

(1) $\frac{1-P(A \cup B)}{P(\overline{B})}$ (2) $\frac{1-P(A \cap B)}{P(\overline{B})}$ (3) $1-P(\overline{A}/B)$ (4) $1-P(A/B)$

11. $\lim\limits_{x \to 2} \frac{x-2}{|x-2|} =$

(1) 0 (2) –2 (3) 1 (4) limit does not exist

12. Let $f(x+y)=f(x)f(y)$ for all x and y. If $f(5)=2$ and $f'(0)=3$, then $f'(5)=$

(1) 6 (2) 5 (3) $\frac{3}{2}$ (4) $\frac{2}{3}$

13. Let $f(x)=\begin{cases} x^2, & x\geq 1 \\ ax+b, & x<1 \end{cases}$, If f is a differentiable function, then :

(1) $a=-1, b=2$

(2) $a=2, b=-1$

(3) $a=-\frac{1}{2}, b=\frac{3}{2}$

(4) $a=\frac{1}{2}, b=\frac{1}{2}$

14. Given the statements :

p : All composite numbers are even numbers.

q : All composite numbers are odd numbers.

Then :

(1) both p and q are true

(2) p is true, q is false

(3) q is true, p is false

(4) both p and q are false

15. The sum of squares of deviations of a set of values is minimum when taken about :

(1) mode

(2) median

(3) geometric mean

(4) arithmetic mean

16. If each observation of a raw data whose variance is σ^2 is multiplied by K, then the variance of the new set is :

(1) $K^2\sigma^2$ (2) $K\sigma^2$ (3) σ^2 (4) $K^2+\sigma^2$

17. A group of 6 boys and 6 girls is randomly divided into two equal groups. The probability that each group contains 3 boys and 3 girls is :

(1) $\frac{90}{231}$ (2) $\frac{100}{231}$ (3) $\frac{110}{231}$ (4) $\frac{36}{231}$

18. In three throws of a pair of dice, the probability of throwing doublets not more than twice is :

(1) $\frac{5}{72}$ (2) $\frac{211}{216}$ (3) $\frac{35}{36}$ (4) $\frac{215}{216}$

19. The probability that a teacher will give an unanounced test during any class meeting is $\frac{1}{5}$. If a student is absent twice, the probability that he will miss atleast one test, is :

(1) $\frac{2}{25}$ (2) $\frac{7}{25}$ (3) $\frac{9}{25}$ (4) $\frac{16}{25}$

20. The probability that the 13th day of a randomly chosen month is a second saturday, is :

(1) $\frac{19}{84}$ (2) $\frac{1}{84}$ (3) $\frac{1}{7}$ (4) $\frac{1}{12}$

21. $\int\frac{dx}{\sqrt{(1-x)(x-2)}}=$

(1) $\sin^{-1}(2x-5)+c$ (2) $\sin^{-1}(3-2x)+c$

(3) $\sin^{-1}(2x-3)+c$ (4) $\sin^{-1}(2x+5)+c$

22. $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx=$

(1) $\sqrt{2}\sin^{-1}(\sin x-\cos x)+c$ (2) $\sqrt{2}\sin^{-1}(\sin x+\cos x)+c$

(3) $\sqrt{2}\cos^{-1}(\sin x-\cos x)+c$ (4) $\sqrt{2}\cos^{-1}(\sin x+\cos x)+c$

23. $\int\frac{\cos 2x-\cos 2\alpha}{\sin x-\sin\alpha}dx=$

(1) $2\sin x-2x\cos\alpha+c$ (2) $2\cos x-2x\sin\alpha+c$

(3) $2\sin x-x\cos\alpha+c$ (4) $2\cos x-x\sin\alpha+c$

24. $\int \frac{xe^x}{(1+x)^2}\,dx =$

(1) $\frac{e^x}{(1+x)^2}+c$ (2) $\frac{e^x}{x+1}+c$ (3) $\frac{-e^x}{x+1}+c$ (4) $\frac{-e^x}{(1+x)^2}+c$

25. Value of $\int_0^{2\pi} \frac{dx}{e^{\sin x}+1}$ is :

(1) 0 (2) $\frac{\pi}{2}$ (3) 2π (4) π

26. Value of $\int_{1/e}^{e} |\log x|\,dx$ is :

(1) $2\left(1-\frac{1}{e}\right)$ (2) $2\left(\frac{1}{e}-1\right)$ (3) $1-\frac{1}{e}$ (4) $\frac{1}{e}-1$

27. The area bounded by $y = x^2$, $y = [x+1]$, $0 \le x < 1$ and the y-axis, where [.] denotes the greatest integer function, is :

(1) $\frac{4}{3}$ sq. units (2) $\frac{1}{3}$ sq. units (3) $\frac{2}{3}$ sq. units (4) $\frac{1}{2}$ sq. units

28. If $I = \int_0^{\pi/2} \frac{dx}{5+3\cos x} = \lambda \tan^{-1}\left(\frac{1}{2}\right)$, then $\lambda =$

(1) $\frac{1}{4}$ (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) 1

29. Area bounded by the parabola $y^2 = 4ax$ and its latus rectum is :

(1) $\frac{2}{3}a^2$ (2) $\frac{5}{8}a^2$ (3) $\frac{4}{3}a^2$ (4) $\frac{8}{3}a^2$

30. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$, the line $x = \sqrt{3}y$ and x-axis is :

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{2\pi}{3}$

31. Let x be the arithmetic mean and y, z be the two geometric means between any two positive numbers. Then $\frac{y^3 + z^3}{xyz} =$

(1) 4 (2) 3 (3) 2 (4) $\frac{3}{2}$

32. If $1 + 6 + 11 + 16 + \ldots\ldots + x = 148$, then $x =$

(1) 31 (2) 26 (3) 41 (4) 36

33. If each term of an infinite G. P. is twice the sum of the terms following it, then the common ratio of the G. P. is :

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$ (3) $\frac{2}{3}$ (4) $\frac{3}{4}$

34. A line joining two points $A(2, 0)$ and $B(3, 1)$ is rotated about A in anticlockwise direction through an angle 15°. The equation of the line in the new position is :

(1) $\sqrt{3}x + y = 2\sqrt{3}$ (2) $\sqrt{3}x - y = 2\sqrt{3}$

(3) $\sqrt{3}y + x = 2\sqrt{3}$ (4) $\sqrt{3}y - x = 2\sqrt{3}$

35. If p_1 and p_2 denote the lengths of the perpendiculars from the origin on the lines $x \sec \alpha + y \operatorname{cosec} \alpha = 2a$ and $x \cos \alpha + y \sin \alpha = a \cos 2\alpha$ respectively, then $\left(\frac{p_1}{p_2} + \frac{p_2}{p_1}\right)^2 =$

(1) $4 \operatorname{cosec}^2 4\alpha$ (2) $4 \sec^2 4\alpha$ (3) $2 \operatorname{cosec}^2 4\alpha$ (4) $2 \cos^2 4\alpha$

36. A line is drawn through the point $P(4, 11)$ to cut the circle $x^2 + y^2 = 9$ at the points A and B. Then $PA \cdot PB =$

(1) 9 (2) 121 (3) 128 (4) 139

37. The focus of the parabola $(y-1)^2 = 12(x-2)$ is :

(1) (5, 1) (2) (1, 5) (3) (2, 1) (4) (3, 0)

38. In an ellipse, if the lines joining a focus to the extremities of the minor axis make an equilateral triangle with the minor axis, the eccentricity of the ellipse is :

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{\sqrt{3}}{2}$

39. The number of lines in three dimensions which are equally inclined to the co-ordinate axes is :

(1) 8 (2) 6 (3) 4 (4) 3

40. $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} =$

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{1}{2}$ (4) 0

41. If $f(x) = \frac{3x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at each point in its domain, then the value of $f(0)$ is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{4}{3}$

42. The number of points of discontinuity for $f(x) = \frac{1}{\log |x|}$, is :

(1) 2 (2) 3 (3) 4 (4) 1

43. If $f(x)=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then $f'(2)=$

(1) $\frac{2}{5}$ (2) $\frac{3}{10}$ (3) $\frac{1}{8}$ (4) $\frac{1}{10}$

44. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w. r. t. $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=0$, is :

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) $\frac{1}{2}$ (4) 1

45. If $x^y=e^{x-y}$, then $\frac{dy}{dx}=$

(1) $\frac{x^2}{(1+\log x)^2}$ (2) $\frac{\log x}{1+\log x}$ (3) $\frac{\log x}{(1+\log x)^2}$ (4) $\frac{x}{(1+\log x)^2}$

46. If $f(x)=x^2e^{-x}$, then the interval in which $f(x)$ increases with respect to x, is :

(1) $(0, 1)$ (2) $(-2, 0)$ (3) $(0, 2)$ (4) $(2, \infty)$

47. The normal to the curve $y = f(x)$ at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with the positive x-axis, then $f'(3)=$

(1) $\frac{3}{4}$ (2) $-\frac{3}{4}$ (3) -1 (4) 1

48. If $f(x) = x(x-2)(x-4)$, $1 \le x \le 4$, then a number satisfying the conditions of the Lagrange's mean value theorem is :

(1) 3 (2) $\frac{4}{3}$ (3) $\frac{5}{2}$ (4) 2

49. If x and y are two real numbers such that $x > 0$ and $xy = 1$, then the minimum value of $x + y$ is :

(1) $\frac{3}{2}$ (2) $\frac{1}{4}$ (3) 1 (4) 2

50. The critical points of the function $f(x) = (2x+1)(x-2)^{2/3}$ are :

(1) 1 and 2 (2) -1 and 2 (3) $-\frac{1}{2}$ and 2 (4) $\frac{1}{2}$ and 2

51. The degree and order of the differential equation of all parabolas whose axis is x-axis, are :

(1) 2, 1 (2) 1, 1 (3) 1, 2 (4) 3, 2

52. A particular solution of $\log\left(\frac{dy}{dx}\right) = 3x + 4y$, $y(0) = 0$, is :

(1) $4e^{3x} - 3e^{-4y} = 12$ (2) $4e^{3x} - 3e^{-4y} = 7$

(3) $4e^{3x} + 3e^{-4y} = 12$ (4) $4e^{3x} + 3e^{-4y} = 7$

53. The solution of the equation $\frac{dy}{dx} = \cos(x - y)$ is :

(1) $x + \cot\left(\frac{x-y}{2}\right) + c$ (2) $x + \tan\left(\frac{x-y}{2}\right) + c$

(3) $y + \cot\left(\frac{x-y}{2}\right) + c$ (4) $y + \tan\left(\frac{x-y}{2}\right) + c$

54. The integrating factor of the differential equation $x \log x \frac{dy}{dx} + y = 2 \log x$, is :

(1) $\log(\log x)$ (2) $\log x$ (3) $-\log x$ (4) e^x

55. A unit vector at $t = 2$ on the curve $x = t^2 + 2$, $y = 4t - 5$, $z = 2t^2 - 6t$, is :

(1) $\frac{1}{3}(2\hat{i} + 2\hat{j} + \hat{k})$

(2) $\frac{1}{3}(2\hat{i} + 2\hat{j} - \hat{k})$

(3) $\frac{1}{\sqrt{3}}(2\hat{i} + 2\hat{j} + \hat{k})$

(4) $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$

56. If $\vec{a}$ and $\vec{b}$ are two unit vectors, then the vector $(\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})$ is parallel :

(1) $\vec{a} + \vec{b}$ (2) $2\vec{a} + \vec{b}$ (3) $\vec{a} - \vec{b}$ (4) $2\vec{a} - \vec{b}$

57. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the three non-coplanar vectors such that $\vec{a} + \vec{b} + \vec{c} = \alpha\vec{d}$ and $\vec{b} + \vec{c} + \vec{d} = \beta\vec{a}$, then $\vec{a} + \vec{b} + \vec{c} + \vec{d} =$

(1) $\vec{0}$ (2) $\alpha\vec{a}$ (3) $\beta\vec{b}$ (4) $(\alpha + \beta)\vec{c}$

58. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then $\left|\frac{\vec{a} - \vec{b}}{2}\right| =$

(1) $2\sin\theta$ (2) $\sin\theta$ (3) $\sin\frac{\theta}{2}$ (4) $2\sin\frac{\theta}{2}$

59. The lines whose vector equations are $\vec{r} = 2\hat{i} - 3\hat{j} + 7\hat{k} + \lambda(2\hat{i} + a\hat{j} + 5\hat{k})$ and $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} - a\hat{j} + a\hat{k})$ are perpendicular for all values of λ and μ if $a =$

(1) 2 (2) 3 (3) 4 (4) 6

60. The number of lines in three dimensions which are equally inclined to the coordinate axes, is :

(1) 2 (2) 4 (3) 6 (4) 8

61. If $f : X \to Y$ defined by $f(x) = \sqrt{3}\sin x + \cos x + 4$ is one-one and onto, then Y is :

(1) $[2, 4]$ (2) $[1, 5]$ (3) $[2, 5]$ (4) $[2, 6]$

62. Inverse of the function $f(x) = \sin^{-1}\left\{4-(x-7)^3\right\}^{1/5}$ is :

(1) $7+\left(4-\sin^5 x\right)^{1/3}$ (2) $7+\left(4+\sin^5 x\right)^{1/3}$

(3) $7-\left(4-\sin^5 x\right)^{1/3}$ (4) $\left(4-\sin^5 x\right)^{1/3}$

63. For real numbers x and y, we write $xRy \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is :

(1) reflexive (2) symmetric

(3) transitive (4) equivalence

64. If $f(x) = \left(a - x^n\right)^{1/n}$, where $a > 0$ and $n \in N$, then $fof(x) =$

(1) a (2) x (3) a^n (4) x^n

65. Value of $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$ is equal to :

(1) $\frac{3}{4}$ (2) $-\frac{1}{4}$ (3) $\frac{5}{8}$ (4) $\frac{1}{16}$

66. If $4\sin^{-1}x + \cos^{-1}x = \pi$, then $x =$

(1) $\frac{3}{4}$ (2) $-\frac{1}{2}$ (3) $\frac{1}{2}$ (4) 1

67. $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) =$

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) $\frac{3\pi}{4}$

68. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is :

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{\pi}{3}$ (4) $\frac{4\pi}{3}$

69. If w is the complex cube root of unity and $A=\begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix}$, then $A^{40}=$

(1) 0 (2) 1 (3) A (4) $-A$

70. The inverse of a skew symmetric matrix of odd order is :

(1) diagonal matrix (2) symmetric matrix

(3) skew-symmetric matrix (4) inverse does not exist

71. If $A(\theta)=\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$ and $AB=I$, then $(\sec^2\theta)B=$

(1) $A(\theta)$ (2) $A(-\theta)$ (3) $-A(\theta)$ (4) $A(\theta/2)$

72. If the matrix $A=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & a \end{bmatrix}$ is unitary, then the value of a is :

(1) -1 (2) 0 (3) -2 (4) 1

73. If $f(\theta)=\begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$, then the value of $f\left(\frac{5\pi}{3}\right)=$

(1) 0 (2) 1 (3) -1 (4) $\frac{\sqrt{3}}{2}$

74. The complex number $z=\begin{vmatrix} 2 & 3+i & -3 \\ 3-i & 0 & -1+i \\ -3 & -1-i & 4 \end{vmatrix}$ is equal to :

(1) $2-5i$ (2) $3-4i$ (3) $5+4i$ (4) None of these

75. If the system of linear equations $x + y + z = 6$, $x + 2y + 3z = 4$, $2x + 5y + \lambda z = k$ has a unique solution, then :

(1) $\lambda = 8, k = 36$ (2) $\lambda = 8, k \neq 36$

(3) $\lambda \neq 8$ (4) $\lambda = 8, k = 24$

76. If x, y, z are all distinct and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then the value of xyz is :

(1) -1 (2) -2 (3) $-\frac{1}{2}$ (4) 1

77. If B is a non-singular matrix and A is square matrix, then $\text{Det}(B^{-1}AB) =$

(1) Det(A) (2) Det(B) (3) $\text{Det}(A^{-1})$ (4) $\text{Det}(B^{-1})$

78. If A is skew-symmetric matrix and n is an even positive integer, then A^n is :

(1) skew-symmetric matrix (2) symmetric matrix

(3) diagonal matrix (4) unitary matrix

79. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2 =$

(1) $2AB$ (2) AB (3) $A + B$ (4) $2(A + B)$

80. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x) =$

(1) $ax(3x + 2a)$ (2) $a(3x + 2a)$ (3) $ax(2x + 3a)$ (4) $x(3x + 2a)$

81. If A, B and C are three sets and X is the universal set such that $n(X) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Then $n(A' \cap B') =$

(1) 200 (2) 300 (3) 400 (4) 500

82. If $A = \left\{4^n - 3n - 1 \mid n \in N\right\}$ and $B = \{9(n-1) \mid n \in N\}$, then $A \cup B =$

(1) N (2) A (3) B (4) $B - A$

83. Let A and B be two non-empty subsets of a set X such that A is not a subset of B, then which of the following is true ?

(1) B is a subset of A

(2) A and B are disjoint

(3) A is a subset of complement of B

(4) A and complement of B are non-disjoint

84. If A and B are two non-empty sets having n elements in common, then the number of elements in common in $A \times B$ and $B \times A$ are :

(1) $2n$ (2) n^2 (3) n^n (4) 2^n

85. A relation R defined from $\{2, 3, 4, 5\}$ to $\{3, 6, 7, 10\}$ by $xRy \Leftrightarrow x$ is relatively prime to y. Then domain of R is :

(1) $\{3, 5\}$ (2) $\{2, 3, 5\}$ (3) $\{2, 3, 4\}$ (4) $\{2, 3, 4, 5\}$

86. Which of the following is a function ?

(1) $\{(x, y) : y = |x|, x, y \in R\}$ (2) $\{(x, y) : x = y^2, x, y \in R\}$

(3) $\{(x, y) : x^2 - y^2 = 1, x, y \in R\}$ (4) $\{(x, y) : x^2 + y^2 = 1, x, y \in R\}$

87. In a triangle ABC, right angled at C and having sides a, b, c, $\tan A + \tan B =$

(1) $\frac{a^2}{bc}$ (2) $\frac{c^2}{ab}$ (3) $\frac{b^2}{ac}$ (4) $\frac{a+b}{c}$

88. If A lies in the second quadrant and $3 \tan A + 4 = 0$, then $2 \cot A - 5 \cos A + \sin A =$

(1) $\frac{5}{3}$ (2) $\frac{7}{10}$ (3) $\frac{23}{10}$ (4) $\frac{37}{10}$

89. If A and B are acute positive angles satisfying the equations $3\sin^2 A + 2\sin^2 B = 1$ and $3 \sin 2A - 2 \sin 2B = 0$, then $A + 2B =$

(1) 0 (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

90. If $\sin 2\theta = \cos 3\theta$ and θ is an acute angle, then $\sin\theta =$

(1) $\frac{\sqrt{5}-1}{4}$ (2) $\frac{\sqrt{5}+1}{4}$ (3) $\frac{\sqrt{3}-1}{2}$ (4) $\frac{\sqrt{3}+1}{4}$

91. If α is an acute angle and $\sin\frac{\alpha}{2} = \frac{\sqrt{x-1}}{2x}$, than $\tan\alpha =$

(1) $\sqrt{\frac{x-1}{x+1}}$ (2) $\frac{\sqrt{x-1}}{x+1}$ (3) $\sqrt{1-x^2}$ (4) $\sqrt{x^2-1}$

92. The statement $P(n)$: "$2 + 4 + 6 + \ldots\ldots + 2n = n(n + 1) + 2$" is given true for $n = k$, then for $n = k + 1$, it is :

(1) not defined (2) true (3) not true (4) meaningless

93. If $iz^3 + z^2 - z + i = 0$, then $|z| =$

(1) 4 (2) 3 (3) 2 (4) 1

94. If $\left|z - \frac{4}{z}\right| = 2$, then the greatest value of $|z|$ is :

(1) $2+\sqrt{2}$ (2) $\sqrt{3}+1$ (3) $\sqrt{5}+1$ (4) $\sqrt{5}-1$

95. Solution of the inequality $\frac{x+1}{x+2} \geq 1$

(1) $(-\infty, -1)$ (2) $(-\infty, -2)$ (3) $(-1, \infty)$ (4) $(2, \infty)$

96. If $0 < r < s \leq n$ and ${}^{n}P_{r} = {}^{n}P_{s}$, then $r + s =$

(1) $2n - 1$ (2) $n - 2$ (3) $2n$ (4) 2

97. The number of words that can be formed by using the letters of the word MATHEMATICS that start as well as end with *A*, is :

(1) 37528 (2) 45360 (3) 90720 (4) 362880

98. A polygon has 44 diagonals. The number of its sides are :

(1) 44 (2) 22 (3) 11 (4) 9

99. The expression $P(x) = \left(\sqrt{x^5 - 1} + x\right)^7 - \left(\sqrt{x^5 - 1} - x\right)^7$ is a polynomial of degree :

(1) 14 (2) 16 (3) 17 (4) 18

100. The remainder when 2^{2003} is divided by 17 is :

(1) 2 (2) 4 (3) 7 (4) 8

Opened for Result Evaluation Purpose

Total No. of Printed Pages : 17

(DO NOT OPEN THIS QUESTION BOOKLET BEFORE TIME OR UNTIL YOU ARE ASKED TO DO SO)

PG-EE-2017

SUBJECT : Mathematics Hons. (Five Year)

C

Sr. No. 10015

Time : **1¼ Hours** Total Questions : **100** Max. Marks : **100**

Roll No. (in figures)__________ (in words) __________

Name __________ Date of Birth __________

Father's Name __________ Mother's Name __________

Date of Exam __________

__________ (Signature of the Candidate)

__________ (Signature of the Invigilator)

SEAL

CANDIDATES MUST READ THE FOLLOWING INFORMATION/INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. ***All questions are compulsory and carry equal marks. The candidates are required to attempt all questions.***
2. The candidates ***must return*** the question booklet as well as OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means/misbehaviour will be registered against him/her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. In case there is any discrepancy in any question(s) in the Question Booklet, the same may be brought to the notice of the Controller of Examinations in writing **within two hours** after the test is over. No such complaint(s) will be entertained thereafter.
4. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
5. ***Use only black or blue ball point pen of good quality in the OMR Answer-Sheet.***
6. There will be ***no negative*** marking. Each correct answer will be awarded ***one*** full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.
7. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. If $f: X \to Y$ defined by $f(x) = \sqrt{3}\sin x + \cos x + 4$ is one-one and onto, then Y is :

(1) $[2, 4]$ (2) $[1, 5]$ (3) $[2, 5]$ (4) $[2, 6]$

2. Inverse of the function $f(x) = \sin^{-1}\left\{4-(x-7)^3\right\}^{1/5}$ is :

(1) $7+\left(4-\sin^5 x\right)^{1/3}$

(2) $7+\left(4+\sin^5 x\right)^{1/3}$

(3) $7-\left(4-\sin^5 x\right)^{1/3}$

(4) $\left(4-\sin^5 x\right)^{1/3}$

3. For real numbers x and y, we write $xRy \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is :

(1) reflexive

(2) symmetric

(3) transitive

(4) equivalence

4. If $f(x) = \left(a - x^n\right)^{1/n}$, where $a > 0$ and $n \in N$, then $fof(x) =$

(1) a (2) x (3) a^n (4) x^n

5. Value of $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$ is equal to :

(1) $\frac{3}{4}$ (2) $-\frac{1}{4}$ (3) $\frac{5}{8}$ (4) $\frac{1}{16}$

6. If $4\sin^{-1}x + \cos^{-1}x = \pi$, then $x =$

(1) $\frac{3}{4}$ (2) $-\frac{1}{2}$ (3) $\frac{1}{2}$ (4) 1

7. $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) =$

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) $\frac{3\pi}{4}$

8. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is :

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{\pi}{3}$ (4) $\frac{4\pi}{3}$

9. If w is the complex cube root of unity and $A=\begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix}$, then $A^{40}=$

(1) 0 (2) 1 (3) A (4) $-A$

10. The inverse of a skew symmetric matrix of odd order is :

(1) diagonal matrix (2) symmetric matrix

(3) skew-symmetric matrix (4) inverse does not exist

11. Let x be the arithmetic mean and y, z be the two geometric means between any two positive numbers. Then $\frac{y^3+z^3}{xyz}=$

(1) 4 (2) 3 (3) 2 (4) $\frac{3}{2}$

12. If $1+6+11+16+ \ldots\ldots +x=148$, then $x=$

(1) 31 (2) 26 (3) 41 (4) 36

13. If each term of an infinite G. P. is twice the sum of the terms following it, then the common ratio of the G. P. is :

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$ (3) $\frac{2}{3}$ (4) $\frac{3}{4}$

14. A line joining two points $A(2, 0)$ and $B(3, 1)$ is rotated about A in anticlockwise direction through an angle 15°. The equation of the line in the new position is :

(1) $\sqrt{3}x+y=2\sqrt{3}$ (2) $\sqrt{3}x-y=2\sqrt{3}$

(3) $\sqrt{3}y+x=2\sqrt{3}$ (4) $\sqrt{3}y-x=2\sqrt{3}$

15. If p_1 and p_2 denote the lengths of the perpendiculars from the origin on the lines $x \sec \alpha + y \operatorname{cosec} \alpha = 2a$ and $x \cos \alpha + y \sin \alpha = a \cos 2\alpha$ respectively, then $\left(\frac{p_1}{p_2}+\frac{p_2}{p_1}\right)^2 =$

(1) $4\operatorname{cosec}^2 4\alpha$ (2) $4\sec^2 4\alpha$ (3) $2\operatorname{cosec}^2 4\alpha$ (4) $2\cos^2 4\alpha$

16. A line is drawn through the point $P(4, 11)$ to cut the circle $x^2+y^2=9$ at the points A and B. Then $PA \cdot PB =$

(1) 9 (2) 121 (3) 128 (4) 139

17. The focus of the parabola $(y-1)^2=12(x-2)$ is :

(1) (5, 1) (2) (1, 5) (3) (2, 1) (4) (3, 0)

18. In an ellipse, if the lines joining a focus to the extremities of the minor axis make an equilateral triangle with the minor axis, the eccentricity of the ellipse is :

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{\sqrt{3}}{2}$

19. The number of lines in three dimensions which are equally inclined to the co-ordinate axes is :

(1) 8 (2) 6 (3) 4 (4) 3

20. $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} =$

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{1}{2}$ (4) 0

21. If A, B and C are three sets and X is the universal set such that $n(X) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Then $n(A' \cap B') =$

(1) 200 (2) 300 (3) 400 (4) 500

22. If $A = \left\{4^n - 3n - 1 \mid n \in N\right\}$ and $B = \{9(n-1) \mid n \in N\}$, then $A \cup B =$

(1) N (2) A (3) B (4) $B - A$

23. Let A and B be two non-empty subsets of a set X such that A is not a subset of B, then which of the following is true ?

(1) B is a subset of A

(2) A and B are disjoint

(3) A is a subset of complement of B

(4) A and complement of B are non-disjoint

24. If A and B are two non-empty sets having n elements in common, then the number of elements in common in $A \times B$ and $B \times A$ are :

(1) $2n$ (2) n^2 (3) n^n (4) 2^n

25. A relation R defined from $\{2, 3, 4, 5\}$ to $\{3, 6, 7, 10\}$ by $xRy \Leftrightarrow x$ is relatively prime
Then domain of R is :

(1) $\{3, 5\}$ (2) $\{2, 3, 5\}$ (3) $\{2, 3, 4\}$ (4) $\{2, 3, 4, 5\}$

26. Which of the following is a function ?

(1) $\{(x, y) : y = |x|, x, y \in R\}$

(2) $\{(x, y) : x = y^2, x, y \in R\}$

(3) $\{(x, y) : x^2 - y^2 = 1, x, y \in R\}$

(4) $\{(x, y) : x^2 + y^2 = 1, x, y \in R\}$

SEAL

27. In a triangle ABC, right angled at C and having sides a, b, c, $\tan A + \tan B =$

(1) $\frac{a^2}{bc}$ (2) $\frac{c^2}{ab}$ (3) $\frac{b^2}{ac}$ (4) $\frac{a+b}{c}$

28. If A lies in the second quadrant and $3 \tan A + 4 = 0$, then $2 \cot A - 5 \cos A + \sin A =$

(1) $\frac{5}{3}$ (2) $\frac{7}{10}$ (3) $\frac{23}{10}$ (4) $\frac{37}{10}$

29. If A and B are acute positive angles satisfying the equations $3\sin^2 A + 2\sin^2 B = 1$ and $3 \sin 2A - 2 \sin 2B = 0$, then $A + 2B =$

(1) 0 (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

30. If $\sin 2\theta = \cos 3\theta$ and θ is an acute angle, then $\sin \theta =$

(1) $\frac{\sqrt{5}-1}{4}$ (2) $\frac{\sqrt{5}+1}{4}$ (3) $\frac{\sqrt{3}-1}{2}$ (4) $\frac{\sqrt{3}+1}{4}$

31. The degree and order of the differential equation of all parabolas whose axis is x-axis, are :

(1) 2, 1 (2) 1, 1 (3) 1, 2 (4) 3, 2

32. A particular solution of $\log\left(\frac{dy}{dx}\right) = 3x + 4y$, $y(0) = 0$, is :

(1) $4e^{3x} - 3e^{-4y} = 12$ (2) $4e^{3x} - 3e^{-4y} = 7$

(3) $4e^{3x} + 3e^{-4y} = 12$ (4) $4e^{3x} + 3e^{-4y} = 7$

33. The solution of the equation $\frac{dy}{dx} = \cos(x - y)$ is :

(1) $x + \cot\left(\frac{x-y}{2}\right) + c$ (2) $x + \tan\left(\frac{x-y}{2}\right) + c$

(3) $y + \cot\left(\frac{x-y}{2}\right) + c$ (4) $y + \tan\left(\frac{x-y}{2}\right) + c$

34. The integrating factor of the differential equation $x \log x \frac{dy}{dx} + y = 2 \log x$, is :

(1) $\log(\log x)$ (2) $\log x$ (3) $-\log x$ (4) e^x

35. A unit vector at $t = 2$ on the curve $x = t^2 + 2$, $y = 4t - 5$, $z = 2t^2 - 6t$, is :

(1) $\frac{1}{3}(2\hat{i} + 2\hat{j} + \hat{k})$ (2) $\frac{1}{3}(2\hat{i} + 2\hat{j} - \hat{k})$

(3) $\frac{1}{\sqrt{3}}(2\hat{i} + 2\hat{j} + \hat{k})$ (4) $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$

36. If $\vec{a}$ and $\vec{b}$ are two unit vectors, then the vector $(\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})$ is parallel :

(1) $\vec{a} + \vec{b}$ (2) $2\vec{a} + \vec{b}$ (3) $\vec{a} - \vec{b}$ (4) $2\vec{a} - \vec{b}$

37. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the three non-coplanar vectors such that $\vec{a} + \vec{b} + \vec{c} = \alpha \vec{d}$ and $\vec{b} + \vec{c} + \vec{d} = \beta \vec{a}$, then $\vec{a} + \vec{b} + \vec{c} + \vec{d} =$

(1) $\vec{0}$ (2) $\alpha \vec{a}$ (3) $\beta \vec{b}$ (4) $(\alpha + \beta) \vec{c}$

38. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then $\left| \frac{\vec{a} - \vec{b}}{2} \right| =$

(1) $2 \sin \theta$ (2) $\sin \theta$ (3) $\sin \frac{\theta}{2}$ (4) $2 \sin \frac{\theta}{2}$

39. The lines whose vector equations are $\vec{r} = 2\hat{i} - 3\hat{j} + 7\hat{k} + \lambda(2\hat{i} + a\hat{j} + 5\hat{k})$ and $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} - a\hat{j} + a\hat{k})$ are perpendicular for all values of λ and μ if $a =$

(1) 2 (2) 3 (3) 4 (4) 6

40. The number of lines in three dimensions which are equally inclined to the coordinate axes, is :

(1) 2 (2) 4 (3) 6 (4) 8

41. $\int \frac{dx}{\sqrt{(1-x)(x-2)}} =$

(1) $\sin^{-1}(2x-5)+c$

(2) $\sin^{-1}(3-2x)+c$

(3) $\sin^{-1}(2x-3)+c$

(4) $\sin^{-1}(2x+5)+c$

42. $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx =$

(1) $\sqrt{2}\sin^{-1}(\sin x-\cos x)+c$

(2) $\sqrt{2}\sin^{-1}(\sin x+\cos x)+c$

(3) $\sqrt{2}\cos^{-1}(\sin x-\cos x)+c$

(4) $\sqrt{2}\cos^{-1}(\sin x+\cos x)+c$

43. $\int \frac{\cos 2x-\cos 2\alpha}{\sin x-\sin\alpha}\,dx =$

(1) $2\sin x-2x\cos\alpha+c$

(2) $2\cos x-2x\sin\alpha+c$

(3) $2\sin x-x\cos\alpha+c$

(4) $2\cos x-x\sin\alpha+c$

44. $\int \frac{xe^x}{(1+x)^2}\,dx =$

(1) $\frac{e^x}{(1+x)^2}+c$ (2) $\frac{e^x}{x+1}+c$ (3) $\frac{-e^x}{x+1}+c$ (4) $\frac{-e^x}{(1+x)^2}+c$

45. Value of $\int_0^{2\pi} \frac{dx}{e^{\sin x}+1}$ is :

(1) 0 (2) $\frac{\pi}{2}$ (3) 2π (4) π

46. Value of $\int_{1/e}^{e} |\log x|\, dx$ is :

(1) $2\left(1-\frac{1}{e}\right)$ (2) $2\left(\frac{1}{e}-1\right)$ (3) $1-\frac{1}{e}$ (4) $\frac{1}{e}-1$

47. The area bounded by $y = x^2$, $y = [x + 1]$, $0 \le x < 1$ and the y-axis, where [.] denotes the greatest integer function, is :

(1) $\frac{4}{3}$ sq. units (2) $\frac{1}{3}$ sq. units (3) $\frac{2}{3}$ sq. units (4) $\frac{1}{2}$ sq. units

48. If $I = \int_{0}^{\pi/2} \frac{dx}{5+3\cos x} = \lambda \tan^{-1}\left(\frac{1}{2}\right)$, then $\lambda =$

(1) $\frac{1}{4}$ (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) 1

49. Area bounded by the parabola $y^2 = 4ax$ and its latus rectum is :

(1) $\frac{2}{3}a^2$ (2) $\frac{5}{8}a^2$ (3) $\frac{4}{3}a^2$ (4) $\frac{8}{3}a^2$

50. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$, the line $x = \sqrt{3}y$ and x-axis is :

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{2\pi}{3}$

51. $\lim_{x \to 2} \frac{x-2}{|x-2|} =$

(1) 0 (2) –2 (3) 1 (4) limit does not exist

52. Let $f(x+y) = f(x)\,f(y)$ for all x and y. If $f(5) = 2$ and $f'(0) = 3$, then $f'(5) =$

(1) 6 (2) 5 (3) $\frac{3}{2}$ (4) $\frac{2}{3}$

53. Let $f(x) = \begin{cases} x^2, & x \geq 1 \\ ax + b, & x < 1 \end{cases}$, If f is a differentiable function, then :

(1) $a = -1, b = 2$

(2) $a = 2, b = -1$

(3) $a = -\frac{1}{2}, b = \frac{3}{2}$

(4) $a = \frac{1}{2}, b = \frac{1}{2}$

54. Given the statements :

p : All composite numbers are even numbers.

q : All composite numbers are odd numbers.

Then :

(1) both p and q are true

(2) p is true, q is false

(3) q is true, p is false

(4) both p and q are false

55. The sum of squares of deviations of a set of values is minimum when taken about :

(1) mode

(2) median

(3) geometric mean

(4) arithmetic mean

56. If each observation of a raw data whose variance is σ^2 is multiplied by K, then the variance of the new set is :

(1) $K^2\sigma^2$ (2) $K\sigma^2$ (3) σ^2 (4) $K^2 + \sigma^2$

57. A group of 6 boys and 6 girls is randomly divided into two equal groups. The probability that each group contains 3 boys and 3 girls is :

(1) $\frac{90}{231}$ (2) $\frac{100}{231}$ (3) $\frac{110}{231}$ (4) $\frac{36}{231}$

58. In three throws of a pair of dice, the probability of throwing doublets not more than twice is :

(1) $\frac{5}{72}$ (2) $\frac{211}{216}$ (3) $\frac{35}{36}$ (4) $\frac{215}{216}$

59. The probability that a teacher will give an unanounced test during any class meeting is $\frac{1}{5}$. If a student is absent twice, the probability that he will miss atleast one test, is :

(1) $\frac{2}{25}$ (2) $\frac{7}{25}$ (3) $\frac{9}{25}$ (4) $\frac{16}{25}$

60. The probability that the 13th day of a randomly chosen month is a second saturday, is :

(1) $\frac{19}{84}$ (2) $\frac{1}{84}$ (3) $\frac{1}{7}$ (4) $\frac{1}{12}$

61. The equation of a plane which passes through (1, –2, 1) and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, is :

(1) $x + 2y + 3 = 0$ (2) $3x + 2y + 1 = 0$

(3) $x - y - 3 = 0$ (4) $x + y + 1 = 0$

62. The foot of the perpendicular for the point (1, 0, 2) to the line $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z+1}{-1}$ is the point :

(1) $\left(\frac{1}{2}, -1, \frac{3}{2}\right)$ (2) $\left(\frac{1}{2}, 1, \frac{3}{2}\right)$ (3) $\left(\frac{1}{2}, 1, \frac{-3}{2}\right)$ (4) $(1, 2, -3)$

63. Which of the following sets is ***not*** convex ?

(1) $\{(x, y) \mid 3x^2 + 2y^2 \le 6\}$ (2) $\{(x, y) \mid 3 \le x^2 + y^2 \le 5\}$

(3) $\{(x, y) \mid x \ge 2, x \le 3\}$ (4) $\{(x, y) \mid y^2 \le x\}$

64. If the constraints in a linear programming problem are changed, then :

(1) the change in constraints is ignored

(2) solution is not defined

(3) the problem is to be re-evaluated

(4) the objective function has to be modified

65. Which of the following statements is *correct* ?

(1) Every LLP admits an optimal solution

(2) A LLP admits a unique optimal solution

(3) The set of all feasible solutions of a LLP is not a convex set

(4) If a LLP admits two optimal solutions then it has an infinite number of optimal solutions

66. If the mean and variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value at most 3 is :

(1) $\frac{4}{5}$ (2) $\frac{7}{8}$ (3) $\frac{15}{16}$ (4) $\frac{13}{16}$

67. A fair coin is tossed n times. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then the value of n is :

(1) 14 (2) 16 (3) 24 (4) 48

68. A letter is known to have come from either CALCUTTA or TATANAGAR. On the envelope, just two consecutive alphabets TA are visible. The probability that letter has come from CALCUTTA is :

(1) $\frac{5}{12}$ (2) $\frac{4}{11}$ (3) $\frac{4}{7}$ (4) $\frac{1}{3}$

69. Three players A, B, C in this order, cut a pack of cards, and the whole pack is reshuffled after each cut. If the winner is one who first draws a diamond, then C's chance of winning is :

(1) $\frac{27}{64}$ (2) $\frac{9}{64}$ (3) $\frac{9}{28}$ (4) $\frac{9}{37}$

70. If two events A and B are such that $P(A) > 0$ and $P(B) \neq 1$, then $P(\overline{A}/\overline{B})$ is equal to :

(1) $\frac{1-P(A \cup B)}{P(\overline{B})}$ (2) $\frac{1-P(A \cap B)}{P(\overline{B})}$ (3) $1-P(\overline{A}/B)$ (4) $1-P(A/B)$

71. If $f(x)=\frac{3x-\sin^{-1}x}{2x+\tan^{-1}x}$ is continuous at each point in its domain, then the value of $f(0)$ is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{4}{3}$

72. The number of points of discontinuity for $f(x)=\frac{1}{\log|x|}$, is :

(1) 2 (2) 3 (3) 4 (4) 1

73. If $f(x)=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then $f'(2)=$

(1) $\frac{2}{5}$ (2) $\frac{3}{10}$ (3) $\frac{1}{8}$ (4) $\frac{1}{10}$

74. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w. r. t. $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=0$, is :

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) $\frac{1}{2}$ (4) 1

75. If $x^y = e^{x-y}$, then $\frac{dy}{dx} =$

(1) $\frac{x^2}{(1+\log x)^2}$ (2) $\frac{\log x}{1+\log x}$ (3) $\frac{\log x}{(1+\log x)^2}$ (4) $\frac{x}{(1+\log x)^2}$

76. If $f(x) = x^2e^{-x}$, then the interval in which $f(x)$ increases with respect to x, is :

(1) $(0, 1)$ (2) $(-2, 0)$ (3) $(0, 2)$ (4) $(2, \infty)$

77. The normal to the curve $y = f(x)$ at the point $(3, 4)$ makes an angle $\frac{3\pi}{4}$ with the positive x-axis, then $f'(3) =$

(1) $\frac{3}{4}$ (2) $-\frac{3}{4}$ (3) -1 (4) 1

78. If $f(x) = x(x - 2)(x - 4)$, $1 \leq x \leq 4$, then a number satisfying the conditions of the Lagrange's mean value theorem is :

(1) 3 (2) $\frac{4}{3}$ (3) $\frac{5}{2}$ (4) 2

79. If x and y are two real numbers such that $x > 0$ and $xy = 1$, then the minimum value of $x + y$ is :

(1) $\frac{3}{2}$ (2) $\frac{1}{4}$ (3) 1 (4) 2

80. The critical points of the function $f(x) = (2x+1)(x-2)^{2/3}$ are :

(1) 1 and 2 (2) -1 and 2 (3) $-\frac{1}{2}$ and 2 (4) $\frac{1}{2}$ and 2

81. If α is an acute angle and $\sin\frac{\alpha}{2} = \frac{\sqrt{x-1}}{2x}$, than $\tan\alpha =$

(1) $\sqrt{\frac{x-1}{x+1}}$ (2) $\frac{\sqrt{x-1}}{x+1}$ (3) $\sqrt{1-x^2}$ (4) $\sqrt{x^2-1}$

82. The statement $P(n)$: "$2 + 4 + 6 + \ldots\ldots + 2n = n(n+1) + 2$" is given true for $n = k$, then for $n = k + 1$, it is :

(1) not defined (2) true (3) not true (4) meaningless

83. If $iz^3 + z^2 - z + i = 0$, then $|z| =$

(1) 4 (2) 3 (3) 2 (4) 1

84. If $\left|z - \frac{4}{z}\right| = 2$, then the greatest value of $|z|$ is :

(1) $2 + \sqrt{2}$ (2) $\sqrt{3} + 1$ (3) $\sqrt{5} + 1$ (4) $\sqrt{5} - 1$

85. Solution of the inequality $\frac{x+1}{x+2} \geq 1$

(1) $(-\infty, -1)$ (2) $(-\infty, -2)$ (3) $(-1, \infty)$ (4) $(2, \infty)$

86. If $0 < r < s \leq n$ and ${}^nP_r = {}^nP_s$, then $r + s =$

(1) $2n - 1$ (2) $n - 2$ (3) $2n$ (4) 2

87. The number of words that can be formed by using the letters of the word MATHEMATICS that start as well as end with A, is :

(1) 37528 (2) 45360 (3) 90720 (4) 362880

88. A polygon has 44 diagonals. The number of its sides are :

(1) 44 (2) 22 (3) 11 (4) 9

89. The expression $P(x) = \left(\sqrt{x^5 - 1} + x\right)^7 - \left(\sqrt{x^5 - 1} - x\right)^7$ is a polynomial of degree :

(1) 14 (2) 16 (3) 17 (4) 18

90. The remainder when 2^{2003} is divided by 17 is :

(1) 2 (2) 4 (3) 7 (4) 8

91. If $A(\theta)=\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$ and $AB=I$, then $(\sec^2\theta)B=$

(1) $A(\theta)$ (2) $A(-\theta)$ (3) $-A(\theta)$ (4) $A(\theta/2)$

92. If the matrix $A=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & a \end{bmatrix}$ is unitary, then the value of a is :

(1) –1 (2) 0 (3) –2 (4) 1

93. If $f(\theta)=\begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$, then the value of $f\left(\frac{5\pi}{3}\right)=$

(1) 0 (2) 1 (3) –1 (4) $\frac{\sqrt{3}}{2}$

94. The complex number $z=\begin{vmatrix} 2 & 3+i & -3 \\ 3-i & 0 & -1+i \\ -3 & -1-i & 4 \end{vmatrix}$ is equal to :

(1) $2-5i$ (2) $3-4i$ (3) $5+4i$ (4) None of these

95. If the system of linear equations $x+y+z=6$, $x+2y+3z=4$, $2x+5y+\lambda z=k$ has a unique solution, then :

(1) $\lambda=8, k=36$ (2) $\lambda=8, k\neq 36$

(3) $\lambda\neq 8$ (4) $\lambda=8, k=24$

96. If x, y, z are all distinct and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}=0$, then the value of xyz is :

(1) –1 (2) –2 (3) $-\frac{1}{2}$ (4) 1

97. If B is a non-singular matrix and A is square matrix, then $\text{Det}\left(B^{-1}AB\right)=$

(1) Det(A) (2) Det(B) (3) $\text{Det}\left(A^{-1}\right)$ (4) $\text{Det}\left(B^{-1}\right)$

98. If A is skew-symmetric matrix and n is an even positive integer, then A^n is :

(1) skew-symmetric matrix (2) symmetric matrix

(3) diagonal matrix (4) unitary matrix

99. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2 =$

(1) $2AB$ (2) AB (3) $A + B$ (4) $2(A + B)$

100. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x) =$

(1) $ax(3x + 2a)$ (2) $a(3x + 2a)$ (3) $ax(2x + 3a)$ (4) $x(3x + 2a)$

Opened for Result Evaluation Purpose. 02/07/2017

Total No. of Printed Pages : 17

(DO NOT OPEN THIS QUESTION BOOKLET BEFORE TIME OR UNTIL YOU ARE ASKED TO DO SO)

PG-EE-2017

SUBJECT : Mathematics Hons. (Five Year)

D

Sr. No. 10008

Time : **1¼ Hours** | Total Questions : **100** | Max. Marks : **100**

Roll No. (in figures)__________ (in words) __________

Name __________ Date of Birth __________

Father's Name __________ Mother's Name __________

Date of Exam __________

(Signature of the Candidate) (Signature of the Invigilator)

SEAL

CANDIDATES MUST READ THE FOLLOWING INFORMATION/INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. ***All questions are compulsory and carry equal marks. The candidates are required to attempt all questions.***
2. The candidates ***must return*** the question booklet as well as OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means/misbehaviour will be registered against him/her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. In case there is any discrepancy in any question(s) in the Question Booklet, the same may be brought to the notice of the Controller of Examinations in writing **within two hours** after the test is over. No such complaint(s) will be entertained thereafter.
4. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
5. ***Use only black or blue ball point pen of good quality in the OMR Answer-Sheet.***
6. There will be ***no negative*** marking. Each correct answer will be awarded ***one*** full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.
7. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. Let x be the arithmetic mean and y, z be the two geometric means between any two positive numbers. Then $\frac{y^3+z^3}{xyz} =$

(1) 4 (2) 3 (3) 2 (4) $\frac{3}{2}$

2. If $1+6+11+16+ \ldots\ldots + x = 148$, then $x =$

(1) 31 (2) 26 (3) 41 (4) 36

3. If each term of an infinite G. P. is twice the sum of the terms following it, then the common ratio of the G. P. is :

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$ (3) $\frac{2}{3}$ (4) $\frac{3}{4}$

4. A line joining two points $A(2, 0)$ and $B(3, 1)$ is rotated about A in anticlockwise direction through an angle 15°. The equation of the line in the new position is :

(1) $\sqrt{3}x + y = 2\sqrt{3}$ (2) $\sqrt{3}x - y = 2\sqrt{3}$

(3) $\sqrt{3}y + x = 2\sqrt{3}$ (4) $\sqrt{3}y - x = 2\sqrt{3}$

5. If p_1 and p_2 denote the lengths of the perpendiculars from the origin on the lines $x \sec \alpha + y \operatorname{cosec} \alpha = 2a$ and $x \cos \alpha + y \sin \alpha = a \cos 2\alpha$ respectively, then $\left(\frac{p_1}{p_2} + \frac{p_2}{p_1}\right)^2 =$

(1) $4\operatorname{cosec}^2 4\alpha$ (2) $4\sec^2 4\alpha$ (3) $2\operatorname{cosec}^2 4\alpha$ (4) $2\cos^2 4\alpha$

6. A line is drawn through the point $P(4, 11)$ to cut the circle $x^2 + y^2 = 9$ at the points A and B. Then $PA \cdot PB =$

(1) 9 (2) 121 (3) 128 (4) 139

7. The focus of the parabola $(y-1)^2 = 12(x-2)$ is :

(1) (5, 1) (2) (1, 5) (3) (2, 1) (4) (3, 0)

8. In an ellipse, if the lines joining a focus to the extremities of the minor axis make an equilateral triangle with the minor axis, the eccentricity of the ellipse is :

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{\sqrt{3}}{2}$

9. The number of lines in three dimensions which are equally inclined to the co-ordinate axes is :

(1) 8 (2) 6 (3) 4 (4) 3

10. $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} =$

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{1}{2}$ (4) 0

11. If $A(\theta) = \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$ and $AB = I$, then $(\sec^2\theta)B =$

(1) $A(\theta)$ (2) $A(-\theta)$ (3) $-A(\theta)$ (4) $A(\theta/2)$

12. If the matrix $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & a \end{bmatrix}$ is unitary, then the value of a is :

(1) -1 (2) 0 (3) -2 (4) 1

13. If $f(\theta) = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$, then the value of $f\left(\frac{5\pi}{3}\right) =$

(1) 0 (2) 1 (3) -1 (4) $\frac{\sqrt{3}}{2}$

14. The complex number $z = \begin{vmatrix} 2 & 3+i & -3 \\ 3-i & 0 & -1+i \\ -3 & -1-i & 4 \end{vmatrix}$ is equal to :

(1) $2-5i$ (2) $3-4i$ (3) $5+4i$ (4) None of these

15. If the system of linear equations $x + y + z = 6$, $x + 2y + 3z = 4$, $2x + 5y + \lambda z = k$ has a unique solution, then :

(1) $\lambda = 8, k = 36$ (2) $\lambda = 8, k \neq 36$

(3) $\lambda \neq 8$ (4) $\lambda = 8, k = 24$

16. If x, y, z are all distinct and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then the value of xyz is :

(1) -1 (2) -2 (3) $-\frac{1}{2}$ (4) 1

17. If B is a non-singular matrix and A is square matrix, then $\text{Det}(B^{-1}AB) =$

(1) $\text{Det}(A)$ (2) $\text{Det}(B)$ (3) $\text{Det}(A^{-1})$ (4) $\text{Det}(B^{-1})$

18. If A is skew-symmetric matrix and n is an even positive integer, then A^n is :

(1) skew-symmetric matrix (2) symmetric matrix

(3) diagonal matrix (4) unitary matrix

19. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2 =$

(1) $2AB$ (2) AB (3) $A+B$ (4) $2(A+B)$

20. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x) =$

(1) $ax(3x+2a)$ (2) $a(3x+2a)$ (3) $ax(2x+3a)$ (4) $x(3x+2a)$

21. The equation of a plane which passes through $(1, -2, 1)$ and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, is :

(1) $x + 2y + 3 = 0$ (2) $3x + 2y + 1 = 0$

(3) $x - y - 3 = 0$ (4) $x + y + 1 = 0$

22. The foot of the perpendicular for the point $(1, 0, 2)$ to the line $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z+1}{-1}$ is the point :

(1) $\left(\frac{1}{2}, -1, \frac{3}{2}\right)$ (2) $\left(\frac{1}{2}, 1, \frac{3}{2}\right)$ (3) $\left(\frac{1}{2}, 1, \frac{-3}{2}\right)$ (4) $(1, 2, -3)$

23. Which of the following sets is ***not*** convex ?

(1) $\{(x, y) \mid 3x^2 + 2y^2 \leq 6\}$ (2) $\{(x, y) \mid 3 \leq x^2 + y^2 \leq 5\}$

(3) $\{(x, y) \mid x \geq 2, x \leq 3\}$ (4) $\{(x, y) \mid y^2 \leq x\}$

24. If the constraints in a linear programming problem are changed, then :

(1) the change in constraints is ignored

(2) solution is not defined

(3) the problem is to be re-evaluated

(4) the objective function has to be modified

25. Which of the following statements is ***correct*** ?

(1) Every LLP admits an optimal solution

(2) A LLP admits a unique optimal solution

(3) The set of all feasible solutions of a LLP is not a convex set

(4) If a LLP admits two optimal solutions then it has an infinite number of optimal solutions

26. If the mean and variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value at most 3 is :

(1) $\frac{4}{5}$ (2) $\frac{7}{8}$ (3) $\frac{15}{16}$ (4) $\frac{13}{16}$

27. A fair coin is tossed n times. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then the value of n is :

(1) 14 (2) 16 (3) 24 (4) 48

28. A letter is known to have come from either CALCUTTA or TATANAGAR. On the envelope, just two consecutive alphabets TA are visible. The probability that letter has come from CALCUTTA is :

(1) $\frac{5}{12}$ (2) $\frac{4}{11}$ (3) $\frac{4}{7}$ (4) $\frac{1}{3}$

29. Three players A, B, C in this order, cut a pack of cards, and the whole pack is reshuffled after each cut. If the winner is one who first draws a diamond, then C's chance of winning is :

(1) $\frac{27}{64}$ (2) $\frac{9}{64}$ (3) $\frac{9}{28}$ (4) $\frac{9}{37}$

30. If two events A and B are such that $P(A) > 0$ and $P(B) \neq 1$, then $P(\overline{A}/\overline{B})$ is equal to :

(1) $\frac{1-P(A \cup B)}{P(\overline{B})}$ (2) $\frac{1-P(A \cap B)}{P(\overline{B})}$ (3) $1-P(\overline{A}/B)$ (4) $1-P(A/B)$

31. $\int \frac{dx}{\sqrt{(1-x)(x-2)}} =$

(1) $\sin^{-1}(2x-5)+c$ (2) $\sin^{-1}(3-2x)+c$

(3) $\sin^{-1}(2x-3)+c$ (4) $\sin^{-1}(2x+5)+c$

32. $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx =$

(1) $\sqrt{2}\sin^{-1}(\sin x - \cos x) + c$

(2) $\sqrt{2}\sin^{-1}(\sin x + \cos x) + c$

(3) $\sqrt{2}\cos^{-1}(\sin x - \cos x) + c$

(4) $\sqrt{2}\cos^{-1}(\sin x + \cos x) + c$

33. $\int \frac{\cos 2x - \cos 2\alpha}{\sin x - \sin \alpha}\, dx =$

(1) $2\sin x - 2x\cos\alpha + c$

(2) $2\cos x - 2x\sin\alpha + c$

(3) $2\sin x - x\cos\alpha + c$

(4) $2\cos x - x\sin\alpha + c$

34. $\int \frac{xe^x}{(1+x)^2}\, dx =$

(1) $\frac{e^x}{(1+x)^2} + c$

(2) $\frac{e^x}{x+1} + c$

(3) $\frac{-e^x}{x+1} + c$

(4) $\frac{-e^x}{(1+x)^2} + c$

35. Value of $\int_0^{2\pi} \frac{dx}{e^{\sin x}+1}$ is :

(1) 0

(2) $\frac{\pi}{2}$

(3) 2π

(4) π

36. Value of $\int_{1/e}^{e} |\log x|\, dx$ is :

(1) $2\left(1-\frac{1}{e}\right)$

(2) $2\left(\frac{1}{e}-1\right)$

(3) $1-\frac{1}{e}$

(4) $\frac{1}{e}-1$

37. The area bounded by $y = x^2$, $y = [x + 1]$, $0 \le x < 1$ and the y-axis, where [.] denotes the greatest integer function, is :

(1) $\frac{4}{3}$ sq. units

(2) $\frac{1}{3}$ sq. units

(3) $\frac{2}{3}$ sq. units

(4) $\frac{1}{2}$ sq. units

38. If $I = \int_0^{\pi/2} \frac{dx}{5+3\cos x} = \lambda \tan^{-1}\left(\frac{1}{2}\right)$, then $\lambda =$

(1) $\frac{1}{4}$ (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) 1

39. Area bounded by the parabola $y^2 = 4ax$ and its latus rectum is :

(1) $\frac{2}{3}a^2$ (2) $\frac{5}{8}a^2$ (3) $\frac{4}{3}a^2$ (4) $\frac{8}{3}a^2$

40. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$, the line $x = \sqrt{3}y$ and x-axis is :

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{2\pi}{3}$

41. If A, B and C are three sets and X is the universal set such that $n(X) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Then $n(A' \cap B') =$

(1) 200 (2) 300 (3) 400 (4) 500

42. If $A = \{4^n - 3n - 1 \mid n \in N\}$ and $B = \{9(n-1) \mid n \in N\}$, then $A \cup B =$

(1) N (2) A (3) B (4) $B - A$

43. Let A and B be two non-empty subsets of a set X such that A is not a subset of B, then which of the following is true ?

(1) B is a subset of A

(2) A and B are disjoint

(3) A is a subset of complement of B

(4) A and complement of B are non-disjoint

44. If A and B are two non-empty sets having n elements in common, then the number of elements in common in $A \times B$ and $B \times A$ are :

(1) $2n$ (2) n^2 (3) n^n (4) 2^n

45. A relation R defined from $\{2, 3, 4, 5\}$ to $\{3, 6, 7, 10\}$ by $xRy \Leftrightarrow x$ is relatively prime to y. Then domain of R is :

(1) $\{3, 5\}$ (2) $\{2, 3, 5\}$ (3) $\{2, 3, 4\}$ (4) $\{2, 3, 4, 5\}$

46. Which of the following is a function ?

(1) $\{(x, y) : y = |x|, x, y \in R\}$ (2) $\{(x, y) : x = y^2, x, y \in R\}$

(3) $\{(x, y) : x^2 - y^2 = 1, x, y \in R\}$ (4) $\{(x, y) : x^2 + y^2 = 1, x, y \in R\}$

47. In a triangle ABC, right angled at C and having sides a, b, c, $\tan A + \tan B =$

(1) $\frac{a^2}{bc}$ (2) $\frac{c^2}{ab}$ (3) $\frac{b^2}{ac}$ (4) $\frac{a+b}{c}$

48. If A lies in the second quadrant and $3 \tan A + 4 = 0$, then $2 \cot A - 5 \cos A + \sin A =$

(1) $\frac{5}{3}$ (2) $\frac{7}{10}$ (3) $\frac{23}{10}$ (4) $\frac{37}{10}$

49. If A and B are acute positive angles satisfying the equations $3\sin^2 A + 2\sin^2 B = 1$ and $3 \sin 2A - 2 \sin 2B = 0$, then $A + 2B =$

(1) 0 (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

50. If $\sin 2\theta = \cos 3\theta$ and θ is an acute angle, then $\sin \theta =$

(1) $\frac{\sqrt{5}-1}{4}$ (2) $\frac{\sqrt{5}+1}{4}$ (3) $\frac{\sqrt{3}-1}{2}$ (4) $\frac{\sqrt{3}+1}{4}$

51. If $f : X \to Y$ defined by $f(x) = \sqrt{3}\,\sin x + \cos x + 4$ is one-one and onto, then Y is :

(1) $[2, 4]$ (2) $[1, 5]$ (3) $[2, 5]$ (4) $[2, 6]$

52. Inverse of the function $f(x) = \sin^{-1}\left\{4 - (x-7)^3\right\}^{1/5}$ is :

(1) $7 + \left(4 - \sin^5 x\right)^{1/3}$ (2) $7 + \left(4 + \sin^5 x\right)^{1/3}$

(3) $7 - \left(4 - \sin^5 x\right)^{1/3}$ (4) $\left(4 - \sin^5 x\right)^{1/3}$

53. For real numbers x and y, we write $xRy \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is :

(1) reflexive (2) symmetric (3) transitive (4) equivalence

54. If $f(x) = \left(a - x^n\right)^{1/n}$, where $a > 0$ and $n \in N$, then $fof(x) =$

(1) a (2) x (3) a^n (4) x^n

55. Value of $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$ is equal to :

(1) $\frac{3}{4}$ (2) $-\frac{1}{4}$ (3) $\frac{5}{8}$ (4) $\frac{1}{16}$

56. If $4\sin^{-1} x + \cos^{-1} x = \pi$, then $x =$

(1) $\frac{3}{4}$ (2) $-\frac{1}{2}$ (3) $\frac{1}{2}$ (4) 1

57. $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) =$

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) $\frac{3\pi}{4}$

58. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is :

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{\pi}{3}$ (4) $\frac{4\pi}{3}$

59. If w is the complex cube root of unity and $A=\begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix}$, then $A^{40}=$

(1) 0 (2) 1 (3) A (4) $-A$

60. The inverse of a skew symmetric matrix of odd order is :

(1) diagonal matrix (2) symmetric matrix

(3) skew-symmetric matrix (4) inverse does not exist

61. If α is an acute angle and $\sin\frac{\alpha}{2}=\frac{\sqrt{x-1}}{2x}$, than $\tan\alpha=$

(1) $\sqrt{\frac{x-1}{x+1}}$ (2) $\frac{\sqrt{x-1}}{x+1}$ (3) $\sqrt{1-x^2}$ (4) $\sqrt{x^2-1}$

62. The statement $P(n)$: "$2+4+6+\ldots\ldots+2n=n(n+1)+2$" is given true for $n=k$, then for $n=k+1$, it is :

(1) not defined (2) true (3) not true (4) meaningless

63. If $iz^3+z^2-z+i=0$, then $|z|=$

(1) 4 (2) 3 (3) 2 (4) 1

64. If $\left|z-\frac{4}{z}\right|=2$, then the greatest value of $|z|$ is :

(1) $2+\sqrt{2}$ (2) $\sqrt{3}+1$ (3) $\sqrt{5}+1$ (4) $\sqrt{5}-1$

65. Solution of the inequality $\frac{x+1}{x+2}\geq 1$

(1) $(-\infty,-1)$ (2) $(-\infty,-2)$ (3) $(-1,\infty)$ (4) $(2,\infty)$

66. If $0 < r < s \le n$ and ${}^nP_r = {}^nP_s$, then $r + s =$

(1) $2n - 1$ (2) $n - 2$ (3) $2n$ (4) 2

67. The number of words that can be formed by using the letters of the word MATHEMATICS that start as well as end with A, is :

(1) 37528 (2) 45360 (3) 90720 (4) 362880

68. A polygon has 44 diagonals. The number of its sides are :

(1) 44 (2) 22 (3) 11 (4) 9

69. The expression $P(x) = \left(\sqrt{x^5 - 1} + x\right)^7 - \left(\sqrt{x^5 - 1} - x\right)^7$ is a polynomial of degree :

(1) 14 (2) 16 (3) 17 (4) 18

70. The remainder when 2^{2003} is divided by 17 is :

(1) 2 (2) 4 (3) 7 (4) 8

71. The degree and order of the differential equation of all parabolas whose axis is x-axis, are :

(1) 2, 1 (2) 1, 1 (3) 1, 2 (4) 3, 2

72. A particular solution of $\log\left(\frac{dy}{dx}\right) = 3x + 4y$, $y(0) = 0$, is :

(1) $4e^{3x} - 3e^{-4y} = 12$ (2) $4e^{3x} - 3e^{-4y} = 7$

(3) $4e^{3x} + 3e^{-4y} = 12$ (4) $4e^{3x} + 3e^{-4y} = 7$

73. The solution of the equation $\frac{dy}{dx} = \cos(x-y)$ is :

(1) $x + \cot\left(\frac{x-y}{2}\right) + c$

(2) $x + \tan\left(\frac{x-y}{2}\right) + c$

(3) $y + \cot\left(\frac{x-y}{2}\right) + c$

(4) $y + \tan\left(\frac{x-y}{2}\right) + c$

74. The integrating factor of the differential equation $x \log x \frac{dy}{dx} + y = 2 \log x$, is :

(1) $\log(\log x)$ (2) $\log x$ (3) $-\log x$ (4) e^x

75. A unit vector at $t = 2$ on the curve $x = t^2 + 2$, $y = 4t - 5$, $z = 2t^2 - 6t$, is :

(1) $\frac{1}{3}(2\hat{i} + 2\hat{j} + \hat{k})$

(2) $\frac{1}{3}(2\hat{i} + 2\hat{j} - \hat{k})$

(3) $\frac{1}{\sqrt{3}}(2\hat{i} + 2\hat{j} + \hat{k})$

(4) $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$

76. If $\vec{a}$ and $\vec{b}$ are two unit vectors, then the vector $(\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})$ is parallel :

(1) $\vec{a} + \vec{b}$ (2) $2\vec{a} + \vec{b}$ (3) $\vec{a} - \vec{b}$ (4) $2\vec{a} - \vec{b}$

77. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the three non-coplanar vectors such that $\vec{a} + \vec{b} + \vec{c} = \alpha \vec{d}$ an $\vec{b} + \vec{c} + \vec{d} = \beta \vec{a}$, then $\vec{a} + \vec{b} + \vec{c} + \vec{d} =$

(1) $\vec{0}$ (2) $\alpha \vec{a}$ (3) $\beta \vec{b}$ (4) $(\alpha + \beta) \vec{c}$

78. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then $\left|\frac{\vec{a} - \vec{b}}{2}\right| =$

(1) $2 \sin \theta$ (2) $\sin \theta$ (3) $\sin \frac{\theta}{2}$ (4) $2 \sin \frac{\theta}{2}$

79. The lines whose vector equations are $\vec{r} = 2\hat{i} - 3\hat{j} + 7\hat{k} + \lambda(2\hat{i} + a\hat{j} + 5\hat{k})$ and $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} - a\hat{j} + a\hat{k})$ are perpendicular for all values of λ and μ if $a =$

(1) 2 (2) 3 (3) 4 (4) 6

80. The number of lines in three dimensions which are equally inclined to the coordinate axes, is :

(1) 2 (2) 4 (3) 6 (4) 8

81. $\lim\limits_{x \to 2} \frac{x-2}{|x-2|} =$

(1) 0 (2) -2 (3) 1 (4) limit does not exist

82. Let $f(x+y) = f(x) f(y)$ for all x and y. If $f(5) = 2$ and $f'(0) = 3$, then $f'(5) =$

(1) 6 (2) 5 (3) $\frac{3}{2}$ (4) $\frac{2}{3}$

83. Let $f(x) = \begin{cases} x^2, & x \geq 1 \\ ax + b, & x < 1 \end{cases}$, If f is a differentiable function, then :

(1) $a = -1, b = 2$
(2) $a = 2, b = -1$
(3) $a = -\frac{1}{2}, b = \frac{3}{2}$
(4) $a = \frac{1}{2}, b = \frac{1}{2}$

84. Given the statements :

p : All composite numbers are even numbers.

q : All composite numbers are odd numbers.

Then :

(1) both p and q are true
(2) p is true, q is false
(3) q is true, p is false
(4) both p and q are false

85. The sum of squares of deviations of a set of values is minimum when taken about :

(1) mode (2) median

(3) geometric mean (4) arithmetic mean

86. If each observation of a raw data whose variance is σ^2 is multiplied by K, then the variance of the new set is :

(1) $K^2\sigma^2$ (2) $K\sigma^2$ (3) σ^2 (4) $K^2+\sigma^2$

87. A group of 6 boys and 6 girls is randomly divided into two equal groups. The probability that each group contains 3 boys and 3 girls is :

(1) $\frac{90}{231}$ (2) $\frac{100}{231}$ (3) $\frac{110}{231}$ (4) $\frac{36}{231}$

88. In three throws of a pair of dice, the probability of throwing doublets not more than twice is :

(1) $\frac{5}{72}$ (2) $\frac{211}{216}$ (3) $\frac{35}{36}$ (4) $\frac{215}{216}$

89. The probability that a teacher will give an unanounced test during any class meeting is $\frac{1}{5}$. If a student is absent twice, the probability that he will miss atleast one test, is :

(1) $\frac{2}{25}$ (2) $\frac{7}{25}$ (3) $\frac{9}{25}$ (4) $\frac{16}{25}$

90. The probability that the 13th day of a randomly chosen month is a second saturday, is :

(1) $\frac{19}{84}$ (2) $\frac{1}{84}$ (3) $\frac{1}{7}$ (4) $\frac{1}{12}$

91. If $f(x)=\frac{3x-\sin^{-1}x}{2x+\tan^{-1}x}$ is continuous at each point in its domain, then the value of $f(0)$ is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{4}{3}$

92. The number of points of discontinuity for $f(x) = \frac{1}{\log|x|}$, is :

(1) 2 (2) 3 (3) 4 (4) 1

93. If $f(x) = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then $f'(2) =$

(1) $\frac{2}{5}$ (2) $\frac{3}{10}$ (3) $\frac{1}{8}$ (4) $\frac{1}{10}$

94. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w. r. t. $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = 0$, is :

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) $\frac{1}{2}$ (4) 1

95. If $x^y = e^{x-y}$, then $\frac{dy}{dx} =$

(1) $\frac{x^2}{(1+\log x)^2}$ (2) $\frac{\log x}{1+\log x}$ (3) $\frac{\log x}{(1+\log x)^2}$ (4) $\frac{x}{(1+\log x)^2}$

96. If $f(x) = x^2 e^{-x}$, then the interval in which $f(x)$ increases with respect to x, is :

(1) (0, 1) (2) (–2, 0) (3) (0, 2) (4) (2, ∞)

97. The normal to the curve $y = f(x)$ at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with the positive x-axis, then $f'(3) =$

(1) $\frac{3}{4}$ (2) $-\frac{3}{4}$ (3) –1 (4) 1

98. If $f(x) = x(x-2)(x-4)$, $1 \le x \le 4$, then a number satisfying the conditions of the Lagrange's mean value theorem is :

(1) 3 (2) $\frac{4}{3}$ (3) $\frac{5}{2}$ (4) 2

99. If x and y are two real numbers such that $x > 0$ and $xy = 1$, then the minimum value of $x + y$ is :

(1) $\frac{3}{2}$ (2) $\frac{1}{4}$ (3) 1 (4) 2

100. The critical points of the function $f(x) = (2x+1)(x-2)^{2/3}$ are :

(1) 1 and 2 (2) -1 and 2 (3) $-\frac{1}{2}$ and 2 (4) $\frac{1}{2}$ and 2

SET : A

1. 2	16. 1	31. 4	46. 3	61. 3	76. 1	91. 4
2. 3	17. 3	32. 1	47. 4	62. 2	77. 3	92. 3
3. 4	18. 3	33. 2	48. 2	63. 4	78. 3	93. 2
4. 2	19. 2	34. 4	49. 3	64. 2	79. 4	94. 3
5. 4	20. 4	35. 4	50. 4	65. 1	80. 2	95. 4
6. 1	21. 3	36. 1	51. 2	66. 3	81. 3	96. 3
7. 2	22. 4	37. 2	52. 1	67. 4	82. 4	97. 1
8. 3	23. 1	38. 4	53. 2	68. 1	83. 1	98. 2
9. 4	24. 2	39. 3	54. 4	69. 4	84. 2	99. 4
10. 1	25. 1	40. 2	55. 3	70. 1	85. 1	100. 1
11. 4	26. 3	41. 4	56. 1	71. 3	86. 3	
12. 2	27. 1	42. 1	57. 1	72. 1	87. 1	
13. 4	28. 4	43. 1	58. 2	73. 2	88. 3	
14. 3	29. 3	44. 2	59. 3	74. 2	89. 4	
15. 2	30. 2	45. 1	60. 1	75. 4	90. 2	

Jitender
02/7/17

Professor & Head,
Department of Mathematics
M.D. University, ROHTAK

1. 4	16. 1	31. 3	46. 3	61. 4	76. 1	91. 4
2. 3	17. 2	32. 4	47. 4	62. 1	77. 1	92. 2
3. 2	18. 4	33. 1	48. 1	63. 1	78. 2	93. 4
4. 3	19. 3	34. 2	49. 4	64. 2	79. 3	94. 3
5. 4	20. 2	35. 1	50. 1	65. 1	80. 1	95. 2
6. 3	21. 3	36. 3	51. 3	66. 3	81. 2	96. 1
7. 1	22. 1	37. 1	52. 4	67. 4	82. 3	97. 3
8. 2	23. 2	38. 4	53. 1	68. 2	83. 4	98. 3
9. 4	24. 2	39. 3	54. 2	69. 3	84. 2	99. 2
10. 1	25. 4	40. 2	55. 1	70. 4	85. 4	100. 4
11. 4	26. 1	41. 3	56. 3	71. 2	86. 1	
12. 1	27. 3	42. 2	57. 1	72. 1	87. 2	
13. 2	28. 3	43. 4	58. 3	73. 2	88. 3	
14. 4	29. 4	44. 2	59. 4	74. 4	89. 4	
15. 4	30. 2	45. 1	60. 2	75. 3	90. 1	

Jitender
02/7/17
Professor & Head,
Department of Mathematics
M.D. University, ROHTAK

SET : C

1. 4	16. 3	31. 3	46. 1	61. 4	76. 3	91. 2
2. 1	17. 1	32. 4	47. 3	62. 3	77. 4	92. 1
3. 1	18. 4	33. 1	48. 3	63. 2	78. 1	93. 2
4. 2	19. 3	34. 2	49. 4	64. 3	79. 4	94. 4
5. 1	20. 2	35. 1	50. 2	65. 4	80. 1	95. 3
6. 3	21. 2	36. 3	51. 4	66. 3	81. 4	96. 1
7. 4	22. 3	37. 1	52. 1	67. 1	82. 2	97. 1
8. 2	23. 4	38. 3	53. 2	68. 2	83. 4	98. 2
9. 3	24. 2	39. 4	54. 4	69. 4	84. 3	99. 3
10. 4	25. 4	40. 2	55. 4	70. 1	85. 2	100. 1
11. 3	26. 1	41. 3	56. 1	71. 3	86. 1	
12. 4	27. 2	42. 1	57. 2	72. 2	87. 3	
13. 1	28. 3	43. 2	58. 4	73. 4	88. 3	
14. 2	29. 4	44. 2	59. 3	74. 2	89. 2	
15. 1	30. 1	45. 4	60. 2	75. 1	90. 4	

Jitender
02/7/17

Professor & Head,
Department of Mathematics,
M.D. University, ROH....

SET : D

1. 3	16. 1	31. 3	46. 1	61. 4	76. 3	91. 3
2. 4	17. 1	32. 1	47. 2	62. 2	77. 1	92. 2
3. 1	18. 2	33. 2	48. 3	63. 4	78. 3	93. 4
4. 2	19. 3	34. 2	49. 4	64. 3	79. 4	94. 2
5. 1	20. 1	35. 4	50. 1	65. 2	80. 2	95. 1
6. 3	21. 4	36. 1	51. 4	66. 1	81. 4	96. 3
7. 1	22. 3	37. 3	52. 1	67. 3	82. 1	97. 4
8. 4	23. 2	38. 3	53. 1	68. 3	83. 2	98. 1
9. 3	24. 3	39. 4	54. 2	69. 2	84. 4	99. 4
10. 2	25. 4	40. 2	55. 1	70. 4	85. 4	100. 1
11. 2	26. 3	41. 2	56. 3	71. 3	86. 1	
12. 1	27. 1	42. 3	57. 4	72. 4	87. 2	
13. 2	28. 2	43. 4	58. 2	73. 1	88. 4	
14. 4	29. 4	44. 2	59. 3	74. 2	89. 3	
15. 3	30. 1	45. 4	60. 4	75. 1	90. 2	

Jitander 02/7/17

Professor & Head,
Department of Mathemat...
M.D. University, ROHT...